STRUCTURE AND MOTION PROBLEMS WITH OCCLUSIONS FOR 1D RETINA VISION

Magnus Oskarsson Kalle Åström Niels Chr. Overgaard

Centre for Mathematical Sciences, Lund University P O Box 118, SE-221 00 Lund, Sweden
magnuso@maths.lth.se, kalle@maths.lth.se, nco@maths.lth.se

ABSTRACT

In this paper we investigate the structure and motion problem for calibrated one-dimensional projections of a two-dimensional environment. In a previous paper the structure and motion problem for all cases with non-missing data was classified and solved. Our aim is here to classify all structure and motion problems, even those with missing data, and to solve them.

1. INTRODUCTION

Understanding of one-dimensional cameras is important in several applications. In [7] it was shown that the structure and motion problem using line features in the special case of affine cameras can be reduced to the structure and motion problem for points in one dimension less, i.e. one-dimensional cameras. Thus solution to 1D structure and motion problems have been used to solve structure and motion problems for lines, [7, 1].

Another area of application is vision for planar motion. It is shown that ordinary vision (two-dimensional retina) can be reduced to that of one-dimensional cameras if the motion is planar, i.e. if the camera is rotating and translating in one specific plane only, cf. [5]. A typical example is the case where a camera is mounted on a vehicle that moves on a flat plane or flat road.

Our personal motivation, however, stems from the *autonomous guided vehicles*, called *AGV*, which are important components for factory automation. The navigation system uses strips of inexpensive reflector tape, *beacons*, which are put on walls or objects along the route of the vehicle, cf. [6] and figure 1.

The *laser scanner* measures the direction from the vehicle to the beacons, but not the distance. This is the information used to calculate the position of the vehicle.

One of the primary vision problems (both 1D and 2D retina) is the so called structure and motion problem. For AGV's this is the procedure to obtain a map of the unknown positions of the beacons using images at unknown positions and orientations.

The overall goal of this work is to solve all solvable structure and motion problems. The purpose of this paper is twofold. Firstly, tools are developed to classify the minimal structure and motion problems with missing data. Secondly, solutions to the structure and motion problem are given for some of these minimal problems.

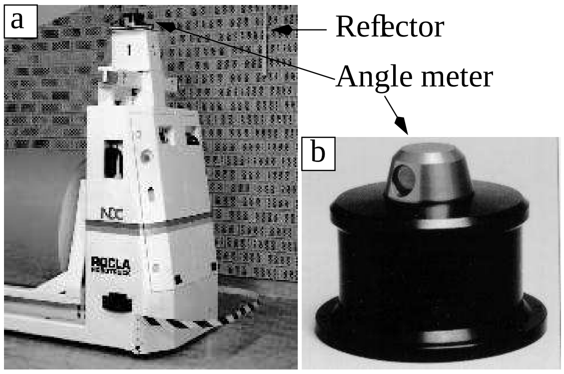

Fig. 1. a: A laser guided vehicle. b: A laser scanner or angle meter.

2. PROBLEM FORMULATION

The one-dimensional vision of a laser scanner can be modeled as a one-dimensional perspective camera (cf. [2] for details).

$$\lambda \mathbf{u} = \mathbf{P}\mathbf{U}\,. \tag{1}$$

Here the camera matrix is calibrated, i.e. it has the following form:

$$\mathbf{P} = \begin{bmatrix} a & b & c \\ -b & a & d \end{bmatrix}, \tag{2}$$

We will often use capital I to denote image number and capital J to denote point number. Thus $\mathbf{u}_{I,J}$ denotes the image direction for point J in image I, $\mathbf{P}_I$ denotes camera matrix for image I and $\mathbf{U}_J$ denotes object point number J.

Motivated by the previous sections the structure and motion problem will now be defined.

Problem 2.1. *Given some of the bearings to n beacons from m different positions $\mathbf{u}_{I,J}$, $(I, J) \in \mathbb{I}$, where $\mathbb{I}$ is an index set representing which beacons J are visible from image* *number* I. *The* **surveying problem** *is to find the depths* $\lambda_{I,J} > 0$, *the reconstructed points* $\mathbf{U}_J$ *and the camera matrices* $\mathbf{P}_I$ *such that*

$$\lambda_{I,J}\mathbf{u}_{I,J} = \mathbf{P}_I\mathbf{U}_J, \qquad \forall(I,J) \in \mathbb{I}\,.$$

In this paper the interest lies in classifying and solving such problems. As such we will consider the problem with both beacons and cameras in general positions. The question whether a structure and motion problem is well-defined or perhaps even over-constrained depends on the structure of the index set $\mathbb{I}$.

In a previous paper [2] we considered only the cases where all beacons are visible in all views. The conclusion there is that the structure and motion problem is well-defined if and only if there are at least 3 views of at least 4 beacons, excluding the case 3 views of 4 beacons.

If it is possible to solve a case with a subset of cameras and beacons, then it is relatively easy to extend that solution to other cameras and points by well known techniques called resection and intersection, [2].

The goal of this paper is to repeat this for the case of missing data. Depending on the index set $\mathbb{I}$ a structure and motion problem can be either **ill-defined** , if there is not, in general, enough data to constrain all unknown variables,**well-defined and minimal**, if there is exactly enough data to constrain the unknown variables (up to a discrete number of solutions), or **well-defined but over-constrained**, if there is more than enough data to constrain the unknown variables.

Some of the minimal cases contain a minimal case as a subproblem. An example of this is the case with four points seen in five images, but where the fourth point is missing from the fifth image. It is minimal, but contains a subproblem (the problem with the first four views only) which is well defined and minimal. We will use the notation **prime problem** for a minimal problem which does not contain a well defined minimal problem as a subproblem. A minimal but not prime problem may in some cases be solved by first solving the contained prime problem and then extend the solution using resection and intersection. In other cases the prime problem may be embedded in the minimal problem in a more complicated manner. We first observe that similar to the case of non-missing data a well-defined but over-constrained problem contains as a subset a problem which is well-defined but minimal. Thus by finding the minimal cases and solving them, we should be able to solve all well-defined problems by extending solutions of minimal problems to the solutions of the original problems.

As the classification is based on the index set $\mathbb{I}$ alone, it is interesting to study these sets. In this paper we consider these sets either as binary matrices, visibility matrices, A of size $m \times n$ where black denotes missing data and white denotes a measurement beacon which is present.

Another way of viewing these index sets is as bi-partite graphs with $m + n$ nodes. There is an edge between node I in the first set and the node J in the second set if the point J is visible in image I. Thus a well-defined minimal case can be considered to be a sub-graph of a well-defined but over-constrained problem.

In the paper we will use the notation $|\mathbb{I}|$ to denote the number of elements in the set $\mathbb{I}$.

3. CLASSIFICATION OF STRUCTURE AND MOTION PROBLEMS

The goal of this section is to give some conditions on what constitutes a well defined minimal problem. From these minimal problems the prime problems can be determined.

3.1. Equivalence classes of index sets

The labeling of the cameras and of the beacons are of no consequence to the structure of the problem under study. Two index sets are considered equivalent if one results from the other by suitable relabelings. This means that there are many surveying problems that have different $\mathbb{I}$ but that correspond in principle to the same problem.

Definition 3.1. An index set $\mathbb{I}$ is said to be of type (m, n, l) if it represents a situation with m images and n points, in which exactly l points are not visible in all of the images, that is, if $|\mathbb{I}| = mn - l$.

Let S_k denote the group of permutations on k symbols. With each permutation $\sigma \in S_k$ is associated a $k \times k$–permutation matrix $(\delta_{i\sigma(j)})$, which will be denoted simply by σ.

Definition 3.2. Two $m \times n$–matrices A and B are said to be *permutation equivalent*, if there exist permutations $\sigma \in S_m$ and $\tau \in S_n$ such that $B = \sigma^T A\tau$. If A and B are permutation equivalent then we write $A \sim B$.

The notion of equivalence of index sets can now be given a formal definition

Definition 3.3. Two index sets $\mathbb{I}$ and $\mathbb{I}'$ are called *equivalent*, and we write $\mathbb{I} \sim \mathbb{I}'$, if their corresponding matrix representations are permutation equivalent.

The relation $\sim$ is easily seen to be an equivalence relation. It follows that $M(m, n, l)$ (or the corresponding index sets) can be partitioned into equivalence classes $M_1, \ldots, M_\omega$ of matrices (or index sets). The number of essentially different index sets is thus seen to be exactly the same as the number $\omega = \omega(m, n, l)$ of equivalence classes. This is the number of principally different problems of type (m, n, l).

Table 1. The number of excess constraints $l = mn - (2n + 3m - 4)$ for the structure and motion problem with m images of n points.

l	n					
m	4	5	6	7	8	9
3	-1	0	1	2	3	4
4	0	2	4	6	8	
5	1	4	7	10		
6	2	6	10			
7	3	8				
8	4					

Table 2. The number ω of different germs for different m and n.

ω	n					
m	4	5	6	7	8	9
3	-	1	1	3	6	11
4	1	3	16	62	225	
5	1	16	155	1402		
6	3	79	1799			
7	6	361				
8	16					

3.2. The germs

A first characterization of a well defined minimal structure and motions problem is that it contains exactly the same number of equations as unknowns. Each object point has two degrees of freedom and each camera state has three. The solution is only defined up to a similarity transformation. This manifold has dimension 4. Using n points and m cameras we thus have $2n+3m-4$ degrees of freedom in the parameters. Each measured bearing gives one constraint on the estimated parameters. Thus for a problem with visibility index set $\mathbb{I}$ we have $|\mathbb{I}|$ equations. This means that minimal problems have $|\mathbb{I}| = 2n+3m-4$. Since the maximum number of equations with m views of n points is mn it is easy to see how many measurements l that have to be occluded to obtain minimal problems, $l = mn - (2n + 3m - 4)$. This number is shown in Table 1.

In order to find the minimal problems we concentrate our efforts on problems of type $(m, n, mn - (2n+3m-4))$.

Definition 3.4. A structure and motion problem of type $(m, n, mn-(2n+3m-4))$ is said to be a *germ* of a minimal problem.

For a structure and motion problem to be minimal and or prime the condition of being a germ is of course only a necessary condition.

We have developed algorithms for calculating the number ω of equivalence classes of germs for different m and n. We have also routines for finding representatives of the different equivalence classes. The description of these algorithms is however beyond the scope of this paper. We have calculated the equivalence classes for some of the first germs. In table 2 the number of distinct germs for these cases are given.

Table 3. The number of minimal configurations for different m and n.

	n					
m	4	5	6	7	8	9
3	-	1	1	2	3	4
4	1	3	12	41	118	
5	1	12	110	876		
6	2	48	1050			
7	3	159				
8	5					

Table 4. The number of prime configurations for different m and n.

	n					
m	4	5	6	7	8	9
3	-	1	0	0	0	0
4	1	1	3	5	8	
5	0	3	22	145		
6	0	6	136			
7	0	0				
8	0					

3.3. Classifying germs

For a given germ the corresponding surveying problem can be minimal or ill-defined. If it is minimal it may or may not be prime. The question of which group a germ belongs to can be categorized in terms of the graph of the index set. If no sub-graph of $\mathbb{I}$ is over determined then the corresponding problem is minimal, and contrarily if the problem is ill-defined at least one sub-graph of $\mathbb{I}$ is over determined. A minimal problem is prime if at least one sub-graph of $\mathbb{I}$ is exactly determined. A first condition that roots out many ill-defined germs is that to every camera there has to be at least three measurements in total otherwise it is impossible to determine the camera uniquely. Similarly there has to be at least two measurements to every point in order to uniquely be able to determine the position of the point. From the germs that fulfill these criteria we try to evaluate which are minimal. For a given germ of type (m, n, l) this can be done in the following manner. For each number $\tilde{m}$ of cameras from 3 to m, we check if the total number of measurement N to the $\tilde{n}$ points which are seen in at least 3 of the cameras is larger than $3\tilde{m} + 2\tilde{n} - 4$. If this is the case for at least one $\tilde{m}$ then the problem is ill-defined, otherwise it is minimal. If the problem is minimal and $N = 3\tilde{m} + 2\tilde{n} - 4$ for some $\tilde{m} < m$ or $\tilde{n} < n$ then the problem is not prime.

We have calculated the number of minimal and prime configurations for some different values of m and n. The results are shown in table 3, where the number of minimal configurations are shown, and in table 4, where the number of prime configurations are shown.

In figure 2a-c and figure 2d-f the prime problems for the configurations of type $(5, 5, 4)$ and $(4, 6, 4)$ are given. Configurations 2a-c seem to be connected to configurations 2d-f. The similarity can be explained using a technique that Carlsson developed in [3, 4].

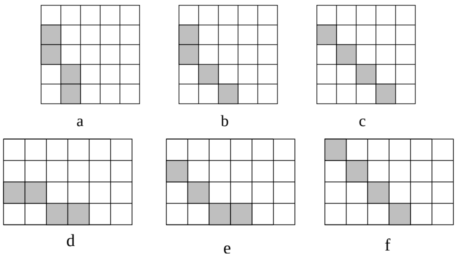

Fig. 2. The 3 distinct configurations for prime cases of type $(5, 5, 4)$ (a-c) and of type $(4, 6, 4)$ (d-f).

Table 5. Some bearing measurements

0.6929	-0.7825	-1.9347	0.3263	-0.6421
0.3206	-0.9479	-1.8732	-0.0041	-0.8289
——	-2.5202	2.4474	-0.9746	-2.3323
2.3024	——	-1.0540	1.8991	0.6499

Theorem 3.1. *The calibrated structure and motion problem with n points and m images is equivalent to the calibrated structure and motion problem with $m+1$ points and $n-1$ images.*

Using this duality one can show that the configurations in figure 2a-c are dual to the configurations in figure 2d-f. If one has the solution to one structure and motion problem the solution to its dual problem can easily be calculated.

4. SOLUTION OF ONE MINIMAL CASE

In section 3.3 we described which the prime problems are. In this section we turn our attention to the task of solving prime problems. There is only one prime configuration for the case of five points in four images. This is the case where one sees five points in two images. In image three, one point is occluded and in image four another point is occluded. All points are visible in camera one and we can use this camera to parameterize the structure. Equation (1) then leads to polynomial equations in the depths of image one. These equations can be solved using resultants and this leads to that there are in general three solutions for this prime problem.

In table 5 bearings for an example of this prime case are shown. The resulting solutions are given in figure 3. In this case there were three real solutions with all depths positive.

5. CONCLUSIONS

In this paper we have begun to classify and solve structure and motion problems for calibrated 1D retina vision with missing data. We have introduced a notation on so called prime structure and motion problems. These are the problems that if solved will allow solutions to all structure and motion problems.

We have begun our work on actually designing algorithms that solve the structure and motion problems for some of these instances. More work is however needed in order to understand and solve these problems. We hope to be able to give more results in this direction in a near future.

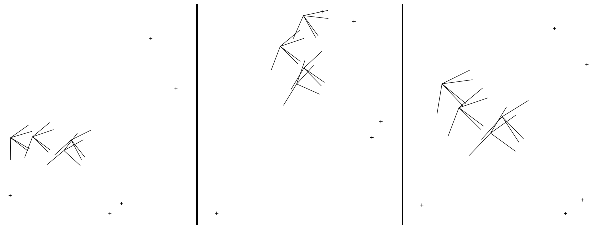

Fig. 3. Three solutions to the minimal case of five points in four images. Beacons are indicated by '+'.

6. REFERENCES

[1] K. Åström, A. Heyden, F. Kahl, and M. Oskarsson. Structure and motion from lines under affine projections. In *Proc. 7th Int. Conf. on Computer Vision, Kerkyra, Greece*, 1999.

[2] K. Åström and M. Oskarsson. Solutions and ambiguities of the structure and motion problem for 1d retinal vision. *Journal of Mathematical Imaging and Vision*, 12:121–135, 2000.

[3] S. Carlsson. Duality of reconstruction and positioning from projective views. In *IEEE Workshop on Representation of Visual Scenes*. IEEE, 1995.

[4] S. Carlsson and D. Weinshall. Dual computation of projective shape and camera positions from multiple images. *Int. Journal of Computer Vision*, 27(3):227–241, 1998.

[5] O. D. Faugeras, L. Quan, and P. Sturm. Self-calibration of a 1d projective camera and its application to the self-calibration of a 2d projective camera. In *Proc. 5th European Conf. on Computer Vision, Freiburg, Germany*, pages 36–52. Springer-Verlag, 1998.

[6] K. Hyyppä. Optical navigation system using passive identical beacons. In *Proceedings intelligent autonomous systems, Amsterdam*, 1987.

[7] L. Quan and T. Kanade. Affine structure from line correspondences with uncalibrated affine cameras. *IEEE Trans. Pattern Analysis and Machine Intelligence*, 19(8):834–845, August 1997.