Motion Estimation in Image Sequences Using the deformation of apparent contours*

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Abstract

In this paper it is shown how to use the generalised epipolar constraint on apparent contours or silhouettes. One such constraint is obtained for each frontier point in each image pair in a sequence of images, to estimate the camera motion.

1. Introduction

...något här...

In [] it was shown how the viewer motion can be calculated from the contraints on the camera motion and the frontier points. However, only a pair of images was considered at the same time. In this situation the problem is often ill-conditioned and therefore it is difficult to accurately estimate the motion parameters. We extend this idea to treat pairs of images simultaneously to obtain more stable results and to recover the full motion of the camera in an image sequence. In this paper, we limit the derivation to the case of an uncalibrated camera with possibly varying intrinsic parameters. It is then well-known that the motion can only be recovered up to a projective transformation, cf []. The generalisation to other cases is straightforward.

Given a sequence of images, the objective is to recover the motion of the camera from the deformation of the apparent contour. In other words, we want to find the camera matrix for each image in the sequence. Considering a pair of camera matrices, the epipolar geometry can be calculated in terms of the fundamental matrix. All frontier points seen from these two views should obey the epipolar constraint and this can be used to find the viewer motion that satisfies all the fundamental matrices of the sequence.

There are other higher-order multilinear constraints between the camera matrices as well. However, in [], it is shown that for $n \geq 4$ images all the multilinear constraints can be generated from the epipolar (bilinear) contraints, so it suffices to consider only the fundamental matrices.

...uppläggning av papper...

The Generalised Epipolar Constraint

... något om surface och dess projection apparent contour...

Consider the camera position at two instants, $P_1 = P(t_1)$, $P_2 = P(t_2)$, and consider all the tangent planes of the surface that go through the two camera centers, cf. Fig . These are called the pencil of **epipolar tangency planes** . In each image, the epipolar tangency planes are projected to a pencil of lines, the **epipolar tangency lines** . They all go through a point, the **epipole e**, each line being tangent to the apparent contour. The tangent points on the apparent contour are called **epipolar tangency points** or **frontier points**. This leads to the following theorem, first formulated in [],

Formulation 1.
Given two images, and the epipoles $\mathbf{e}_1$ and $\mathbf{e}_2$, the pencil of lines through $\mathbf{e}_1$ in image one, which are tangent to an apparent contour, and the corresponding pencil of epipolar tangency lines in image two are projectively related.

This is the generalised epipolar constraint. After introducing coordinate systems in both images, it can also be expressed by the fundamental matrix, F, and the corresponding frontier points in image one $w_{1,i}$ resp. in image two $w_{2,i}$,

$$w_{1,i}^T F w_{2,i} = 0, \quad i = 1, ..., m, \tag{1}$$

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where m is the number of frontier points.

This highlights the similarities to the well-known epipolar constraint for points, cf. []. Notice that the two epipoles $\mathbf{e}_1$ and $\mathbf{e}_2$ can be obtained as the left and right nullspace of the fundamental matrix, ie. $\mathbf{e}_1^T\mathbf{F} = 0$ and $\mathbf{F}\mathbf{e}_2 = 0$. The frontier points can be obtained from the epipolar tangency constraint, ie.

$$\det\begin{bmatrix}\mathbf{e}_1 & w_{1,i} & (w_{1,i})_s\end{bmatrix} = 0$$
$$and$$
$$\det\begin{bmatrix}\mathbf{e}_2 & w_{2,i} & (w_{2,i})_s\end{bmatrix} = 0,$$

where subscript s denotes differention with respect to the parametrisation of the apparent contours. Thus, we are interested in computing two quantities, the fundamental matrix and the frontier points. If the fundamental matrix is known, the frontier points can be calculated, and vice versa.

The Maximum Likelihood Estimate

... inledningsmening om MLE.. The method is standard, cf. [].

We want to estimate the camera motion for a sequence of n images. Let $\mathbf{P}$ be an abstract variable for the camera matrices $P_i = P(t_i), \quad t_1 < ... < t_n$, describing the motion. As residuals α_i for the maximum likelihood estimate, we use the epipolar tangency constraint in equation (1), ie.

$$\alpha_i = w_{q,i}^T F_q w_{q,i}. \tag{2}$$

where w_i, w_i are corresponding frontier points and F_q are fundamental matrices. We are dealing with $n > 1$ images, and thus we have $\binom{n}{2}$ different fundamental matrices, $F_q, q = 1, \dots, \binom{n}{2}$. Since the frontier points are calculated independently, it is a reasonable assumption that the residuals α_i are independent and Gaussian with zero-mean and standard deviation σ_i. Then, we find the maximum likelihood by minimising the following function

$$g(\mathbf{P}) = Y^T Y \tag{3}$$

where $Y = (Y_1, \dots, Y_m)^T = (\alpha_1/\sigma_1, \dots, \alpha_m/\sigma_m)^T$.

The choice of residuls α_i according to (2) has several advantages. First of all, in [] it was reported that this parametrisation gives very accurate estimates of the fundamental matrix for points. Since the error in α_i are mostly due to localisation erros of the frontier points, the standard deviation σ_i is easily estimated using first order approximation by

$$hejhopp.$$

Each F_q is a function of two camera matrices, ie.

$$bla...$$

The problem is clearly non-linear and in order to minimise g in (3) over the motion parameter manifold, we have chosen the method of Levenberg-Marquardt, see []. Then, the first and second derivatives of g with respect to $\mathbf{P}$ needs to be calculated. The first derivate of $g(x)$ is

$$hejhopp$$

and

$$\frac{\frac{\partial\alpha_i}{\partial x}\sigma_i - \alpha_i\frac{\partial\sigma_i}{\partial x}}{\sigma_i^2}.$$

and

$$\frac{\partial\alpha_i}{\partial x} = ...$$

Hej hopp $f_i = \int_{t=0}^{\infty}$ formel.

$$f_i = \int_{t=0}^{\infty}$$

$$f_i = \int_{t=0}^{\infty} \tag{4}$$

kalle Åström enligt (4) se även [1].

2. Experimental Validation on Simulated Data

3. Conclusions

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References

[1] A. Heyden. Reconstruction from three images of six point objects. In *Proc. Symposium on Image Analysis, SSAB, Halmstad, Sweden*, pages 57–66, 1994.