Nonlinear Long-Wave Deformation and Runup in a Basin of Varying Depth

Ira Didenkulova

Institute of Cybernetics, Tallinn, Estonia; ira@cs.ioc.ee

Nonlinear shallow water theory

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+g\frac{\partial \eta}{\partial x}=0 \qquad \frac{\partial \eta}{\partial t}+\frac{\partial}{\partial x}\left[(h+\eta)u\right]=0$$

η is the water level displacement,
u is the horizontal velocity of water flow,
g is a gravity acceleration and
h is unperturbed water depth assumed to be constant

basin of a slowly varying depth $dh/dx \sim \alpha \to 0$

Nonlinear wave transformation in a basin of slowly varying depth

$$U(t,y)=U_0\left[t-V(U)y\right] \qquad V(U)=\frac{3}{2}\frac{U}{gh_0}$$

For the initial condition $U(t,y=0)=U_0(t)$

$$Y_{Br}=\frac{1}{\max(-dV_0/dt)}=\frac{2gh_0}{3\omega a}$$

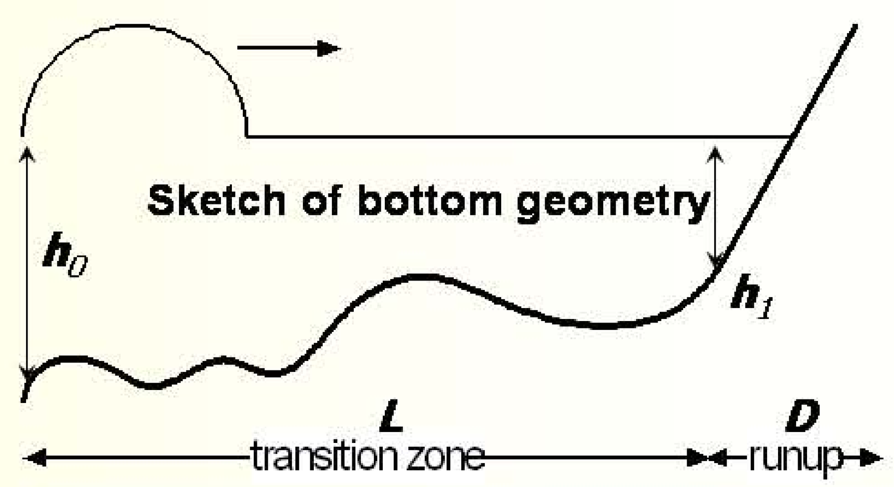
distance of the first wave breaking

For the initial sine wave with "velocity" amplitude *a* and frequency ω

$$\eta(t,x)=\left(\frac{h_0}{h(x)}\right)^{1/4}\eta_0\left(t-\tau(x)+\frac{3\eta y(x)}{2h_0\sqrt{gh_0}}\left(\frac{h(x)}{h_0}\right)^{1/4}\right) \qquad y(x)=\int\left(\frac{h_0}{h(x)}\right)^{7/4}d$$

η_0 is an initial shape of water displacement

Snapshot of tsunami approaching to the coast (Indian Ocean, December 26, 2004)



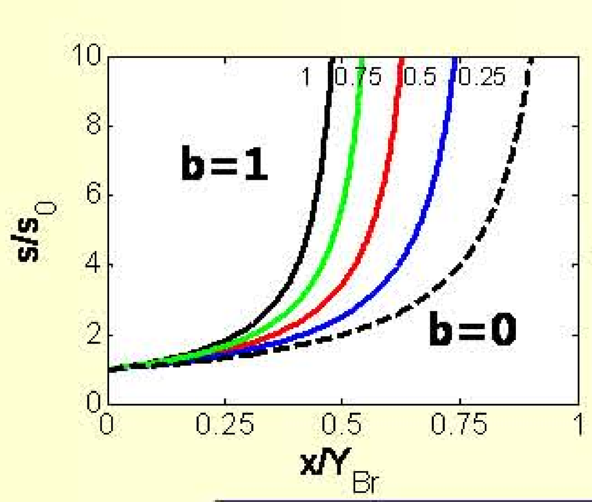
$$s=\max(\partial\eta/\partial x)=\left(\frac{h_0}{h}\right)^{3/4}\frac{s_0}{1-y/Y_{Br}}$$

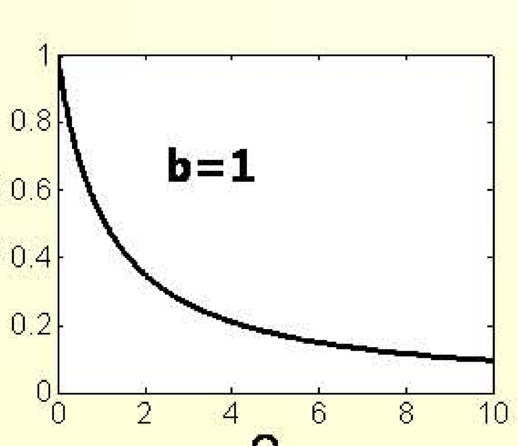
face-slope steepness

Examples of wave deformation for various bottom profiles

$$h(x)=h_0-(h_0-h_1)\left(\frac{x}{L}\right)^b$$

$$y(x)=\frac{x\left[(3b-4)\left(1-\frac{h_0-h_1}{h_0}\left(\frac{x}{L}\right)^b\right)^{3/4}{}_2F_1\left(\frac{1}{b},\frac{3}{4},1+\frac{1}{b},\frac{h_0-h_1}{h_0}\left(\frac{x}{L}\right)^b\right)+4\right]}{3b\left(1-\frac{h_0-h_1}{h_0}\left(\frac{x}{L}\right)^b\right)^{3/4}}+const$$





$$\frac{X^1_{Br}}{Y_{Br}}=\frac{1}{Q_1}\left[1-\left(1+\frac{3}{4}Q_1\right)^{-4/3}\right] \qquad Q_1=\frac{h_0-h_1}{h_0}\frac{Y_{Br}}{L}=\beta\frac{Y_{Br}}{h_0}$$

$$Q_{4/3}=\frac{h_0-h_1}{h_0}\left(\frac{Y_{Br}}{L}\right)^{4/3} \qquad \frac{X_{Br}^{4/3}}{Y_{Br}}=\frac{1}{(1+Q_{4/3})^{3/4}}$$

b=4/3

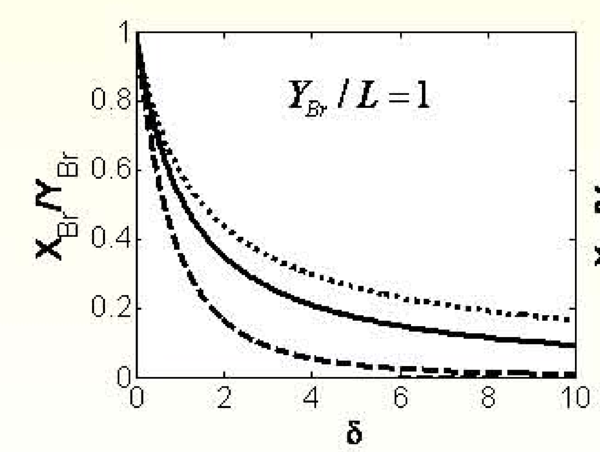
$$\left(3Q_{1/2}^2+32\right)\left(1-Q_{1/2}\sqrt{\frac{X_{Br}}{Y_{Br}}}^{\,1/2}\right)^{3/4}=8\left(4-3Q_{1/2}\sqrt{\frac{X_{Br}}{Y_{Br}}}^{\,1/2}\right)$$

$$Q_{1/2}=\frac{h_0-h_1}{h_0}\sqrt{\frac{Y_{Br}}{L}}$$

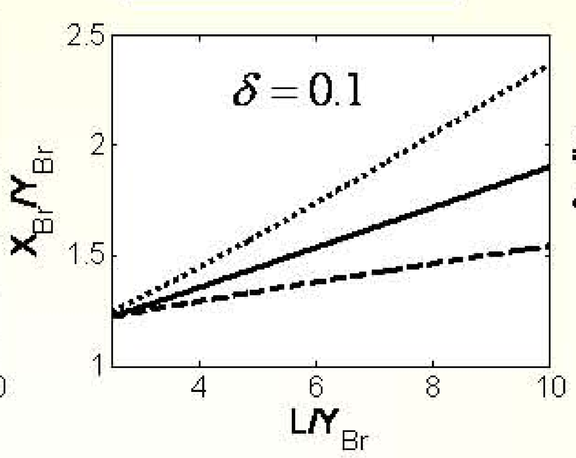
b=1/2

Comparison between different beach profiles

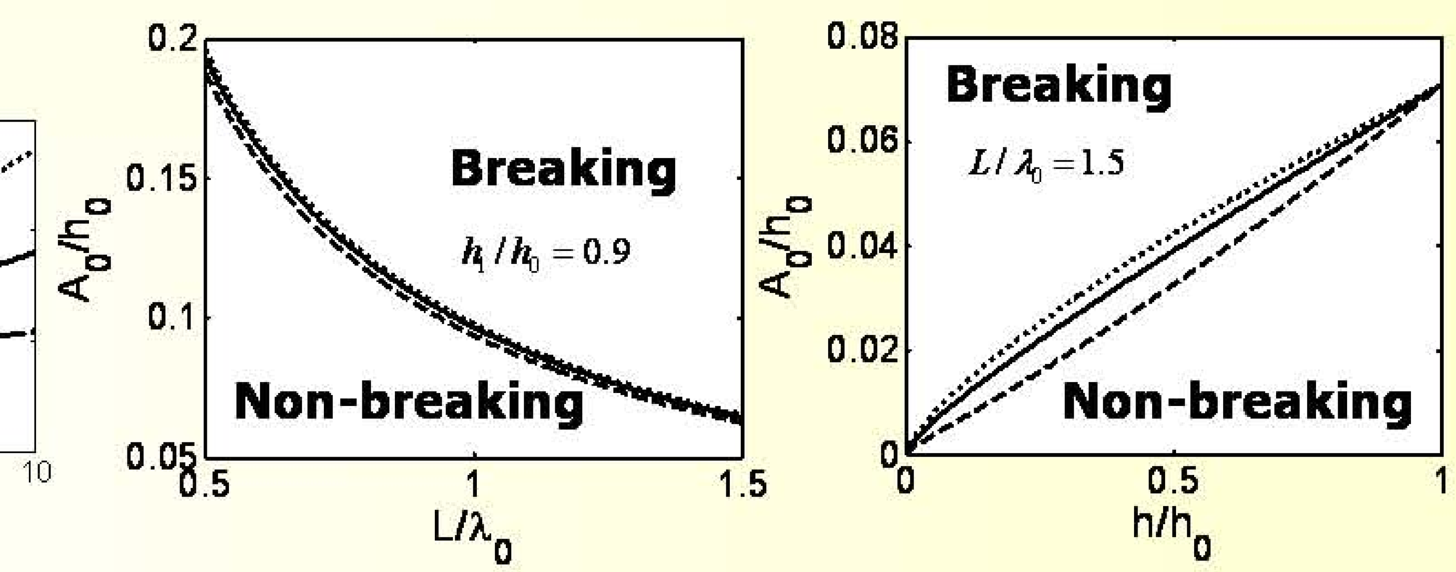
$$\delta=(h_1-h_0)/h_0$$



Breaking distance plotted against depth variation



Breaking distance and the ratio Y_{Br}/L



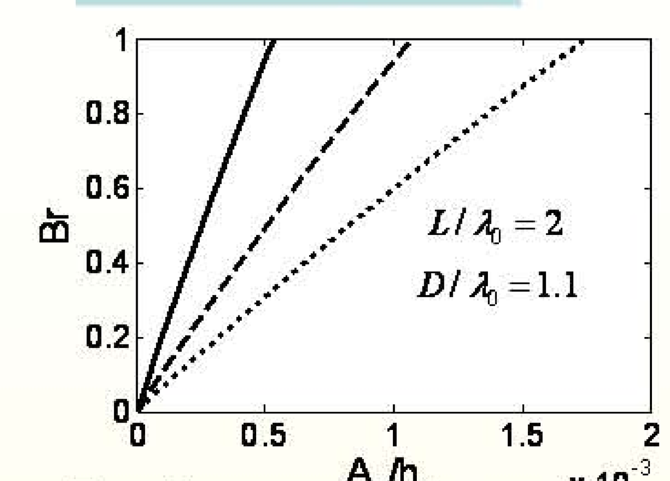
Breaking and non-breaking regimes of the wave transformation in the transition zone

Plane beach (solid line), convex (dotted) and concave (dashed) bottom profiles

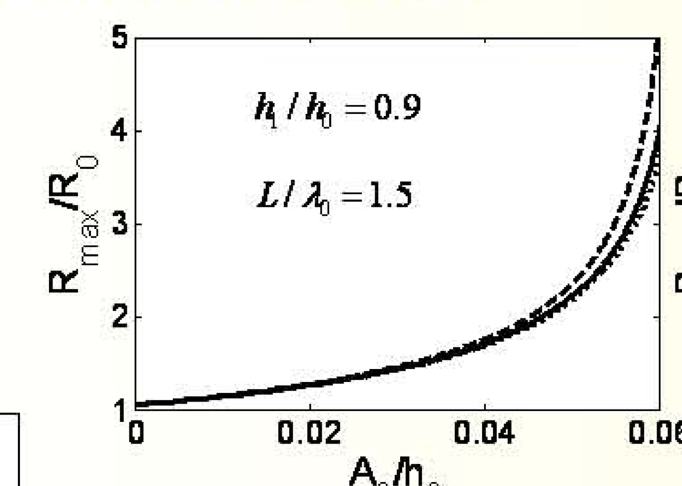
Runup of deformed waves on a coast

$$\eta(t)=H_0\,\mathrm{Re}\sum_{n=1}B_n\exp(in\omega t)$$

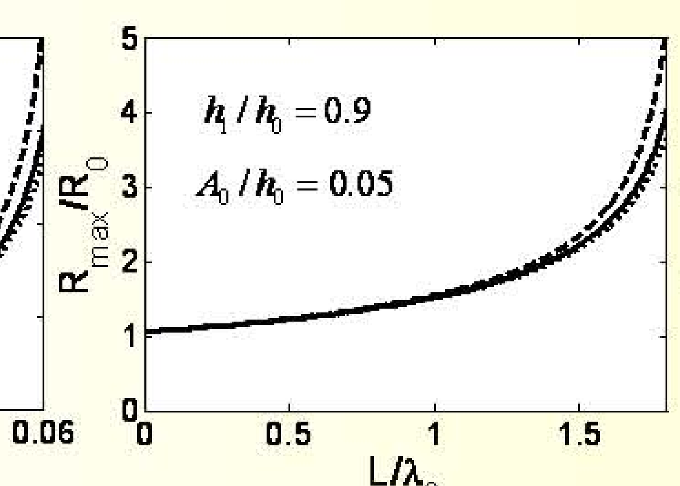
$$R_{max}=2\pi H_0\sqrt{\frac{2D}{\lambda}\frac{s}{s_0}}$$



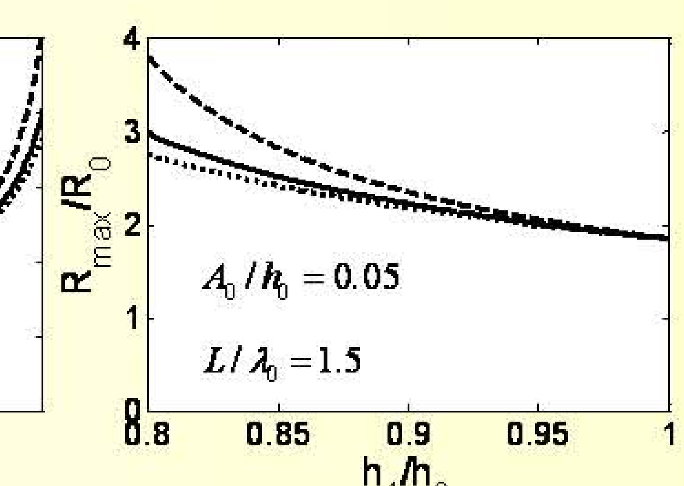
Breaking parameter versus initial wave amplitude; , **solid, dashed and dotted lines corresponds to** h_1/h_0=0.7; 0.8; 0.9



Maximum runup height plotted against the amplitude of an initial wave



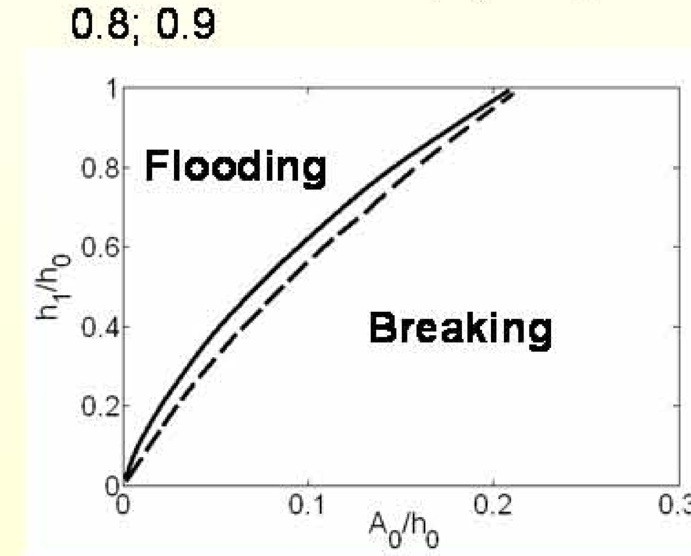
Maximum runup height plotted against the width of transmission zone



Maximum runup height plotted against the depth ratio

$$Br=(2\pi)^3\frac{H_0}{h_1}\sqrt{2\left(\frac{D}{\lambda}\right)^5}\max\mathrm{Re}\sum_{n=1}n^{5/2}B_n\exp\left[i\left(n\omega t+\frac{5\pi}{4}\right)\right]$$

Breaking parameter at the runup stage



Various scenarios of the wave runup on a coast (dashed line corresponds to a plane beach and solid line to a concave beach)

The problem of long wave shoaling and runup on the coast is described in the framework of 1D nonlinear shallow-water theory. The bottom geometry considered in the paper consists of the transition zone with a slowly varying depth (in the scale of the wavelength) and a beach of constant slope in the vicinity of the shoreline. An incident wave in the open ocean has a sinusoidal shape and small amplitude. In the transition zone, the wave shoaling is described by an asymptotic solution in the form of the quasi-Riemann wave with varying amplitude. The wave steepness and spectral components are calculated for three bottom profiles: a beach of constant slope, a convex and a concave profiles. It is shown that a concave profile is more "nonlinear" as its average depth along the wave path is smaller than for other profiles. The wave runup on a plane beach is studied in the framework of the Carrier-Greenspan approach with a nonlinear deformed wave in the transition zone as an input wave. The runup characteristics and the condition of wave breaking are analyzed with respect to the wave parameters in the open ocean. It is demonstrated that wave steepness is the most significant parameter characterizing the runup process. The comparison between different transition zones shows that in the general case, the concave beach gives a larger increase in the wave steepness and greater amplification of the wave amplitude. At the same time waves propagating along such a beach break sooner.

Ira Didenkulova. Nonlinear Long-Wave Deformation and Runup in a Basin of Varying Depth // Nonlinear Processes in Geophysics, 16(1) 23-32 (2009).