Achim Schroll, October 16, 2009

LUND UNIVERSITY
Centre for Mathematical Sciences

FMN135: Adaptive FE Methods

Assignment 2

Duality based error estimation of output functionals

- **Task 1 (Exercise 3.1):** Functional oriented a posteriori error estimates can also be stated in terms of energy–norm error bounds. Consider the Poisson model problem. With the primal and dual errors $e := u - u_h$ and $e^* := z - z_h$, respectively, there holds

$$|J(e)| = (\nabla e, \nabla z) = (\nabla e, \nabla e^*) \leq \|\nabla e\| \|\nabla e^*\| \ .$$

  Then, any a posteriori bound for the energy–norm error supplies also a bound for $J(e)$. Specify a situation in which this simple minded approach is ineffective. Why is this approach not suited to extract refinement indicators from the error estimate?

- **Task 2 (Exercise 3.3a):** Consider the Poisson problem

$$-\Delta u = 1 \quad \text{in} \Omega \ , \quad u|_{\delta\Omega} = 0 \ ,$$

  on the domain $\Omega = (-1,1)^2 \backslash (-0.5, 0.5)^2$. Compute the function values $u(a)$ and $\delta_1 u(a)$ in the point $a = (0.75, 0.75)$. Use the provided tutorial program `step-14` with dual, weighted error estimation and mesh optimization strategy. Compute for a sequence of increasing degrees of freedom $100 \leq dof \leq 20000$ and monitor the estimated errors.