Achim Schroll, September 7, 2009

LUND UNIVERSITY

FMN135: Adaptive FE Methods

Centre for Mathematical Sciences

Assignment 1

Duality based error estimation

Consider a system of linear equations $Ax = b$ and its discrete version $A_h x_h = b_h$. The goal ist to estimate a linear error functional

$$J(e) = (e, j) \ , \quad j \in \mathbb{R}^n \ .$$

It holds that $|J(e)| \leq \sum_i |\rho_i||z_i|$, where $\rho = b - Ax_h$ is the residual and z is the dual solution $A^* z = j$. In practice however, also z is unknown and needs to be computed fom a discrete dual system $A_h^* z_h = j_h$.

Task: Repeat the arguments from Chap. 1.4 to show that

$$|J(e)| \leq \sum_i |\rho_i| \cdot |z_{h,i}| + \|\rho\| \cdot \|z - z_h\| \ .$$

Interprete the inequality. What qualities do the individual terms describe?

Remark: The assignment is mainly ment as a repetition and warm up. Don't be surprised that the derivation actually is quite short.