Achim Schroll, November 6, 2008

LUND UNIVERSITY

Finite Volume Methods
Centre for Mathematical Sciences

Assignment 3

Topics: 2D Riemann problems in gas dynamics
Deadline: Friday, Dec. 19th. 2008

Background: Godunov type finite volume methods rely heavily on numerical approximations of Riemann problems. In one space dimension a large variety of Riemann solvers is available. In the early 90s there where attempts to build truly multidimensional finite volume methods based on numerical approximations of multidimensional Riemann problems. In his PhD thesis (1993 at ETH Zürich), Carsten Schulz–Rinne classified 15 genuinely different configurations for the Riemann problem for polytropic gas in two space dimension. In this assignment you shall experience the complexity of 2D Riemann problems by numerical simulation.

Assignment: Consider the Euler system for polytropic gas in the plane $\mathbb{R}^2$. A Riemann problem is an initial value problem with piecewise constant initial data. We consider piecewise constant data in the four quadrants, separated by the coordinate system. With such data solutions will be self–similar, that is they are invariant under scaling $(t, x, y) \mapsto (\alpha t, \alpha x, \alpha y)$ and actually depend on the similarity variables $\xi = x/t$ and $\eta = y/t$ only. A reasonable setting is to cover the square $[-3, 3]^2$ by a 400×400 mesh and to simulate the solution over two time units $t \in [0, 2]$. Obviously the boundaries are artificial and it is appropriate to specify (non–reflecting) outflow conditions by zero–order extrapolation (LeVeque 7.2.1). The initial data in the following configurations is constructed such that each discontinuity initially, before interaction, defines one single wave, a shock wave or a contact discontinuity respectively.

"Configuration 3" featuring four shock waves is defined as follows:

1. quadrant: $p_1 = \rho_1 = 1.5$, $u_1 = v_1 = 0.0$.
2. quadrant: $p_2 = 0.3$, $\rho_2 = \rho_1 \left(\frac{p_2}{p_1} + \frac{\gamma-1}{\gamma+1}\right) / \left(1 + \frac{\gamma-1}{\gamma+1}\frac{p_2}{p_1}\right)$,
   $u_2 = u_1 + \sqrt{(p_2 - p_1)(\rho_2 - \rho_1)/(\rho_1\rho_2)}$, $v_2 = v_1$.
3. quadrant: $p_3 = 0.029$, $\rho_3 = 0.138$,
   $u_3 = u_2$, $v_3 = v_1 + \sqrt{(p_2 - p_1)(\rho_2 - \rho_1)/(\rho_1\rho_2)}$.
4. quadrant: $p_4 = p_2$, $\rho_4 = \rho_2$, $u_4 = u_1$, $v_4 = v_3$.

"Configuration F" involving two shocks and two contacts is given by:

1. quadrant: $p_1 = 0.4$, $\rho_1 = \left(\gamma - 1 + \frac{\gamma-1}{\gamma+1}\right) / \left(1 + (\gamma-1)^2/(\gamma+1)\right)$, $u_1 = v_1 = 0.0$.
2. quadrant: $p_2 = \rho_2 = 1.0$, $u_2 = u_1 + \sqrt{(p_2 - p_1)(\rho_2 - \rho_1)/(\rho_1 \rho_2)}$, $v_2 = 0.0$.
3. quadrant: $p_3 = 1.0$, $\rho = 0.8$, $u_3 = v_3 = 0.0$.
4. quadrant: $p_4 = \rho_4 = 1.0$, $u_4 = 0.0$, $v_4 = \sqrt{(p_4 - p_1)(\rho_4 - \rho_1)/(\rho_1 \rho_4)}$.

Study both configurations by numerical simulation. Report contour plots of density and Mach number, and evtl pressure. Try to find the sonic line, for example. Identify some of the wave types and/or interactions you observe.

Literature:

Carsten W. Schulz–Rinne: Classification of the Riemann problem fro two–dimensional gas dynamics, SIAM J. Math. Anal. 24, (1993) 76–88.

Carsten W. Schulz–Rinne, James P. Collins and Harland M. Glaz: Numerical solution of the Riemann problem for two–dimensional gas dynamics, SIAM J. Sci. Comput. 14 (1993) 1394–1414.