1. True or false:
   (a) The Discrete Cosine Transform of a set of real data may be complex
   (b) The DCT of a sum of two vectors is the sum of the two separate DCT (dct(v+w) = dct(v) + dct(w))
   (c) The DCT can be computed with a fast algorithm of complexity $n \log_2 n$.

2. Can a function of the form $f(x) = c_1 + c_2 \cos(\pi x)$ interpolate data points $(0, 1)$ and $(2, 1/2)$? If so, write the matrix equation that must be solved to find the coefficients of the interpolating function. Otherwise, justify your answer.

3. Give two applications for the DCT.

4. What are the normal equations for solving least squares approximation with the DCT?

5. How can you get a compression of 4:1 on a file containing 1 048 576 data points?

6. Which type of transformation is used in JPEG compression?

7. True or false: The DFT implies a periodic extension of the function defined over a finite interval, and the DCT implies a periodic even extension of the function.

8. What kind of basis functions should you use to approximate a periodic odd function with real values?

9. True or false: The discrete cosine transform is a linear transformation that is not necessarily invertible.

10. What is the difference between compressing by taking the average and by quantization?

11. True or false: Quantization and Huffman coding are examples of lossy compression.

12. Explain why it is good practice to subtract 128 from an 8 x 8 image matrix before compressing.

13. Draw a Huffman tree for the message DO THE PROBLEMS ASSIGNED, including spaces, and convert to Huffman coding. Compute the Shannon information and compare.
14. Translate the transformed, quantized image components

\[
\begin{pmatrix}
-4 & -1 & 1 & -1 & 0 & 1 & 0 & 0 \\
3 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

to bit streams, using JPEG Huffman coding.