

# Analysis of Computational Algorithms for Linear Multistep Methods

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## Abstract

Linear multistep methods (LMMs) constitute a class of time-stepping methods for the solution of initial value ODEs; the most well-known methods of this class are the Adams methods (AMs) and the backward differentiation formulae (BDFs). For the fixed stepsize LMMs there exists an extensive error and stability analysis; in practical computations, however, one always uses a variable stepsize. It is well known that the varying stepsize affects the error and stability properties of the LMMs; the analysis of the variable stepsize LMMs is generally much more complicated than for the fixed stepsize methods. Due to this, many of the computational algorithms in LMM codes are based on results from the analyses for fixed stepsize LMMs or onestep methods and the purpose of these algorithms is partly to supply operating conditions that resemble as much as possible the properties for the fixed stepsize methods.

In this monograph we investigate different variable stepsize AMs and BDFs, with regard to method representation, solution of nonlinear equations, error estimation, and stepsize control. The methods are thoroughly derived and presented under a uniform taxonomy. In the error analysis the approach is to avoid premature Taylor approximations and crude norm bounds to be able to reveal some general properties of error propagation and error estimates for the different variable stepsize AMs and BDFs. The stepsize control is viewed from a control theoretical standpoint, where we, opposed to the conventional analysis, also take the errors' dependence on past stepsizes into account.

The results show that the assumptions, on which the conventional strategies rely, are not always fulfilled and, furthermore, that they can yield some undesirable secondary effects. We show that the predictors may have a severe impact on the behaviour of both method and error estimation properties. This will not only affect the choice of methods and predictors, but also several stages of the computational algorithms.