LUND UNIVERSITY

Centre for Mathematical Sciences
Mathematics, Faculty of Science

Written Examination
Ordinary Differential Equations II
Saturday, April 1, 2017
08.00-13.00

Note: Only students who are registered or re-registered on the course are allowed to take the exam.

No aids allowed. Use the distributed paper sheets and write only on one side. Fill in the cover sheet completely. Write legibly (in Swedish or English). Motivate your conclusions clearly and concisely; draw a picture if appropriate.

Test results: *Posted Monday, April 2, before 17.00.*

Oral exams: *Wednesday, April 5 – Friday, April 7. State your preference (day and AM/PM) on the cover sheet of your test – at least two options.*

1. Find all fixed points of the system

$$\begin{cases} x' = x - y^2 - z^2, \\ y' = 2x^2 - xy^2 - 1, \\ z' = -z. \end{cases}$$

Determine whether they are stable, asymptotically stable or unstable.

2. Show that the boundary value problem

$$\begin{aligned} &y''(x) + 9y(x) = f(x), \quad 0 < x < \pi, \\ &y(0) = 1, \\ &y'(\pi) = -1, \end{aligned}$$

has a unique solution for all $f \in C[0, \pi]$. Express the solution using Green's function.

3. Show that the system

$$\begin{cases} x' = 3x + y - 2xe^{x^2+2y^2} \\ y' = -x + 3y - 2ye^{x^2+2y^2} \end{cases}$$

has a regular periodic orbit.

4. Consider the system

$$\begin{cases} x' = x(1 - x - y), \\ y' = 2 - ye^x. \end{cases}$$

Sketch the phase portrait for $x, y \geq 0$. Then prove that any solution $(x(t), y(t))$ with $x(0) \geq 0$ and $y(0) \geq 0$ remains in the closed first quadrant for $t \geq 0$ and converges to the fixed point $(0, 2)$ as $t \to \infty$.

Please, turn over!

5. Consider the initial value problem

$$x'(t) = t + e^{-t}e^{x}, \quad x(0) = 0.$$

Show that solution blows up in finite time, that is, there exists a finite $T > 0$ such that $x(t)$ is defined on the interval $[0, T)$ and $x(t) \to \infty$ as $t \to T^{-}$.

Hint: It might be useful to first show that $x(t) > t$ for $t > 0$.