LUND
UNIVERSITY

Centre for Mathematical Sciences
Mathematics, Faculty of Science

Written Examination
Ordinary Differential Equations II
Monday, May 5, 2014
08.00-13.00

Note: Only students who are registered or re-registered on the course are allowed to take the exam.

No aids allowed. Use the distributed paper sheets and write only on one side. Fill in the cover sheet completely and write your initials on each sheet. Write legibly (in Swedish or English). Motivate your conclusions clearly and concisely; draw a picture if appropriate.

Test results: *Posted Wednesday, May 7, before 17.00.*

Oral exams: *Monday, May 12 – Tuesday, May 13. State your preference (day and AM/PM) on the cover sheet of your test.*

1. Find all fixed points of the system

$$\begin{cases} x' = (x^2 - y)(y^2 - 1) \\ y' = x^3 - xy + y^2 - 1 \end{cases}$$

and determine their stability properties.

2. Prove that the origin is an asymptotically stable fixed point for the autonomous system corresponding to the equation $x'' + (x')^3 + x^3 = 0$.

3. For which $b > 0$ does the problem

$$\begin{aligned} &y'' - 2y' + 2y = f, \quad 0 < x < b \\ &y(0) = y(b) = 0, \end{aligned}$$

have a unique solution for all $f \in C[0, b]$? Express the solution using Green's function when it is possible.

4. Prove that the system

$$\begin{cases} x' = x + y - x^3 \\ y' = -x + y - y^3 \end{cases}$$

has a non-constant, periodic solution.

5. Consider the equation

$$-u'' + q(x)u = 0, \quad x > 0,$$

where $q \in C([0, \infty), \mathbb{R})$. Assume that $q(x) \to q_\infty$ as $x \to \infty$ and let u be a non-trivial solution.

a) Show that u has infinitely many zeros if $q_\infty < 0$ and at most finitely many zeros if $q_\infty > 0$.

b) Show that both alternatives are possible if $q_\infty = 0$.