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1. a) The problem is unbounded.

b) Let A, P, M, and S denote the volume (in hl) of apple, pear, mixed and standard cider, respectively. The problem is then to

$$\begin{array}{ll} \text{maximize} & z = 196A + 210P + 280M + 442S \\ \text{subject to} & \left\{ \begin{array}{rcrcrcrcl} 1.6A &+& 1.8P &+& 3.2M &+& 5.4S &\leq& 80, \\ 1.2A &+& 1.2P &+& 1.2M &+& 1.8S &\leq& 40, \\ -0.8A &+& 0.2P &+& 0.2M &+& 0.2S &\leq& 0, \\ -0.3A &+& 0.7P &-& 0.3M &-& 0.3S &\leq& 0, \\ && && A, \; P, & M, & S &\geq& 0. \end{array} \right. \end{array}$$

2. $x_1 = 2$, $x_2 = 1$ and $z = 1$.

3. a) The transportation problem is

$$\begin{array}{ll} \text{maximize} & z = \sum_{i,j} c_{ij} x_{ij} \\ \text{subject to} & \left\{ \begin{array}{rcll} \sum_{j=1}^{n} x_{ij} &=& s_i, & i = 1, \ldots, m, \\ \sum_{i=1}^{m} x_{ij} &=& d_j, & j = 1, \ldots, n, \\ x_{ij} &\geq& 0, & i = 1, \ldots, m, \; j = 1, \ldots, n. \end{array} \right. \end{array}$$

Let $x_{ij} = s_i d_j / S$. Then clearly $x_{ij} \geq 0$ since s_i, d_j and S are all positive. Moreover,

$$\sum_{j=1}^{n} x_{ij} = \sum_{j=1}^{n} \frac{s_i d_j}{S} = \frac{s_i}{S} \sum_{j=1}^{n} d_j = s_i,$$
$$\sum_{i=1}^{m} x_{ij} = \sum_{i=1}^{m} \frac{s_i d_j}{S} = \frac{d_j}{S} \sum_{i=1}^{m} s_i = d_j,$$

and so x_{ij} satisfies all the constraints of the problem.

b) Assume that $X = [x_{ij}]$ is a feasible solution. Then clearly, by the last constraint, $x_{ij} \geq 0$. Also, $\sum_{j=1}^{n} x_{ij} = s_i$ and $x_{ij} \geq 0$ implies that $x_{ij} \leq s_i$ for all i, j. Likewise, $\sum_{i=1}^{m} x_{ij} = d_j$ together with $x_{ij} \geq 0$ implies that $x_{ij} \leq d_j$ for all i, j.

4. Job number 1, 2, 3, 4, 5, 6 is assigned to person number 4, 5, 3, 1, 6, 2, respectively.

5. a) The shortest path is $1 \to 3 \to 5 \to 8 \to 9$, and the distance is 19.

b) The algorithm complexity is $\mathcal{O}(n^2)$.

6. The LP problem has variables x_{ij}, $i, j = 1, \ldots, 12$ and t, and is formulated as

$$
\begin{array}{ll}
\text{minimize} & z = t \\
\text{subject to} & \left\{ \begin{array}{rcll}
-x_{ij} - t & \leq & -r_{ij}, & i, j = 1, \ldots, 12, \\
x_{ij} - t & \leq & r_{ij}, & i, j = 1, \ldots, 12, \\
\sum_{j=1}^{12} x_{ij} & = & p_i, & j = 1, \ldots, 12, \\
\sum_{i=1}^{12} x_{ij} & = & q_j, & i = 1, \ldots, 12, \\
x_{ij} & \geq & 0, & i, j = 1, \ldots, 12.
\end{array} \right.
\end{array}
$$