

INGA HJÄLPMEDEL/NO TOOLS ARE ALLOWED.

Lösningarna skall vara försedda med ordentliga motiveringar. Skriv dina lösningar på svenska eller engelska.

Careful explanations of your solutions should be provided. Write your solutions in Swedish or English.

1. a) Solve the problem

$$\begin{array}{ll} \text{maximize} & z = x_1 + x_2 + 3x_3 \\ \text{subject to} & \left\{ \begin{array}{l} x_1 + 2x_2 - x_3 = 1, \\ x_2 - x_3 \leq 3, \\ x_1, x_2, x_3 \geq 0, \end{array} \right. \end{array}$$

with the simplex method. Explain how you choose incoming and outgoing variables, and motivate how you can deduce that you have found an optimum or that the problem is unbounded or infeasible. (0.5)

b) A cider company produces four types of cider: Apple, Pear, Mixed and Standard. Every hectoliter of each type of cider requires a certain number of working hours p for production, and a certain number of hours q for packaging. Also the profit v (in units of SEK/hectoliter of cider sold) made for each of these ciders is different depending on the type. These numbers p , q and v for the four types of ciders are specified below:

Cider type	p	q	v
Apple	1.6	1.2	196
Pear	1.8	1.2	210
Mixed	3.2	1.2	280
Standard	5.4	1.8	442

In a week the cider company can spend 80 hours on production and 40 hours on packaging. Also, the company has decided that the Apple cider shall constitute at least 20% of the total volume of cider produced, while the Pear cider shall constitute at most 30% of the total volume of cider produced.

The company wants to decide how much of each sort of cider it should produce in a week so as to maximize its profit under the constraints described above. Formulate this as a linear programming problem on standard form. (0.5)

VAR GOD VÄND! / PLEASE TURN OVER!

2. Consider the following ILP problem:

$$\begin{array}{ll} \text{maximize} & z = x_1 - x_2 \\ \text{subject to} & \begin{cases} 3x_1 - x_2 \leq 5, \\ x_1 + x_2 \leq 3, \\ x_1, x_2 \geq 0 \text{ integers.} \end{cases} \end{array}$$

Solve this problem using the cutting plane method. Explain how you construct the equations for the cutting planes. Use the dual system for solving the new system(s).

3. Consider the general transportation problem in which

$$S = \sum_{i=1}^m s_i = \sum_{j=1}^n d_j.$$

a) Show that $x_{ij} = s_i d_j / S$ is a feasible solution. (0.5)

b) Show that if $X = [x_{ij}]$ is a feasible solution, then, for all i and j , $0 \leq x_{ij} \leq \min(s_i, d_j)$. (0.5)

4. Solve the assignment (minimization) problem with the following cost matrix:

$$\begin{bmatrix} 3 & 5 & 4 & 2 & 8 & 4 \\ 8 & 3 & 6 & 2 & 4 & 3 \\ 4 & 4 & 2 & 8 & 3 & 5 \\ 3 & 8 & 7 & 4 & 9 & 7 \\ 7 & 3 & 9 & 3 & 3 & 5 \\ 3 & 7 & 4 & 5 & 5 & 8 \end{bmatrix}.$$

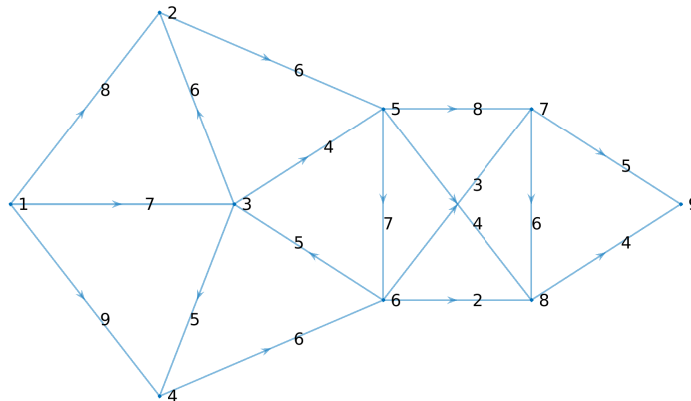
Make sure that you describe what you do in each step of the algorithm.

5. Dijkstra's algorithm for finding the shortest distance from node s to node t in a network with distances c_{ij} is described below, using the notation in the table:

D	The set of unreached nodes (at the current step),
y_j	Node price of node j ,
p_j	Predecessor index of node j ,
v_j	Temporary node price of node j ,
q_j	Temporary predecessor index of node j .

1. Let $D = N \setminus \{s\}$, $y_s = 0$ and $v_j = \infty$ (or a large number M) for all $j \in D$. Let $k = s$.
2. For every $j \in D$ such that $(k, j) \in E$, if $y_k + c_{kj} < v_j$ let $q_j = k$ and $v_j = y_k + c_{kj}$.
3. Replace k by the first index $j \in D$ which has the smallest v_j . Let $y_k = v_k$, $p_k = q_k$ and replace D by $D \setminus \{k\}$.
4. If $t \notin D$, stop.
5. Go to 2.

a) Below is a network which describes a map with distances connecting 9 places.



Find the shortest distance from node $1 = s$ to node $9 = t$ in the network, using Dijkstra's shortest path algorithm as described above. Make sure that you write down the updated values for k , y_k , p_k , v_j , q_j and D for each iteration, and answer with an optimal path and the shortest distance. (0.5)

b) Compute the algorithm complexity of Dijkstra's algorithm for a (general) network with n nodes, by using the algorithm outline above.

Hint: You need to estimate the maximum number of iterations, and the maximum number of arithmetic operations in each iteration. (0.5)

6. In a certain city, there is a subway line with 12 stations. One year ago, a careful survey of the number of commuters between different pairs of stations was made. In particular, for each pair (i, j) with $i \neq j$ and $i, j \in \{1, \dots, 12\}$, the average number r_{ij} of commuters per day that use the subway to go between station i to station j (that is, enter at station i and exit at station j) was recorded. As one year has passed since this survey, it is reasonable to expect that these numbers r_{ij} have changed, since many people have changed their residence or place of work in the meantime. So one would like to update this survey. But we don't want to repeat the careful survey done earlier. So suppose we do the following now: for every $i \in \{1, \dots, 12\}$ we record the average number p_i of commuters per day that enter the subway at station i , and we record also the average number q_i of commuters per day that leave the subway at station i . Now we want to replace the old numbers r_{ij} with new numbers x_{ij} that are consistent with the observations p_i and q_j , while differing "as little as possible" from the old numbers r_{ij} . Formulate this as a linear programming problem on standard form. Take $\max_{i,j} |x_{ij} - r_{ij}|$ as a measure of how much the numbers x_{ij} differ from the numbers r_{ij} .

LYCKA TILL! / GOOD LUCK!