LUNDS TEKNISKA HÖGSKOLA
MATEMATIK

TENTAMENSSKRIVNING
LINJÄR OCH KOMBINATORISK OPTIMERING
2018-03-16

INGA HJÄLPMEDEL/NO TOOLS ARE ALLOWED.
Lösningarna skall vara försedda med ordentliga motiveringar. Skriv dina lösningar på svenska eller engelska.
Careful explanations of your solutions should be provided. Write your solutions in Swedish or English.

1. We would like to solve the problem

$$\begin{array}{ll} \text{maximize} & z = x_1 + 3x_2 - 2x_3 \\ \text{subject to} & \begin{cases} 2x_1 + 3x_2 - x_3 \leq 2, \\ x_1 - 4x_2 = -5, \\ 7x_1 - 5x_2 - 2x_3 \leq -4, \\ x_1, \quad x_2, \quad x_3 \geq 0, \end{cases} \end{array}$$

with the two-phase method.

a) Write down a LP problem on canonical form that should be solved in phase 1. What is the first simplex tableau for phase 1? (0.4)

b) Make one iteration of the simplex method starting from the tableau in subproblem a). Explain how you choose incoming and outgoing variables. (0.2)

c) After some more iterations, you find that the final tableau of phase 1 is

	x_1	x_2	x_3	x_4	x_5	y_1	y_2	$\mathbf{x_b}$
x_2	$-1/4$	1	0	0	0	$1/4$	0	$5/4$
x_5	$1/4$	0	0	-2	1	$11/4$	-1	$23/4$
x_3	$-11/4$	0	1	-1	0	$3/4$	0	$7/4$
z	0	0	0	0	0	1	1	0

where y_1 and y_2 are the artificial variables. (Don't worry if you have used a different number of artificial variables in subproblem a) and b).) Does the original problem have a feasible solution? If so, what should the first tableau of phase 2 be? (0.2)

d) Solve the LP problem with the simplex method, starting from the tableau you found in subproblem c). Is there an optimal solution? If so, write it down. If the problem is unbounded, explain how you can see this from the tableau. (0.2)

VAR GOD VÄND! / PLEASE TURN OVER!

2. Consider the following ILP problem:

$$\begin{array}{ll} \text{maximize} & z = x_1 - x_2 \\ \text{subject to} & \begin{cases} 3x_1 - x_2 \leq 5, \\ x_1 + x_2 \leq 3, \\ x_1,\ x_2 \geq 0 \text{ integers.} \end{cases} \end{array}$$

In this exercise, you will solve this problem geometrically, using the branch and bound method.

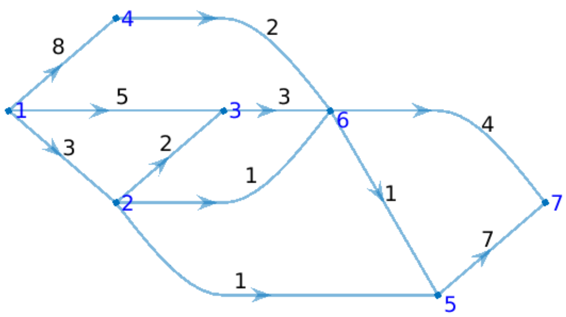
a) Draw the feasible set of the LP relaxed problem (i.e. with the integer constraint removed) in the x_1x_2-plane, together with some level curves of the objective function. Does the LP relaxed problem have an optimal solution? If so, write it down together with the optimal value. (0.5)

b) Solve the ILP problem with the branch and bound method without the simplex method, i.e. each LP problem should be solved geometrically as in subproblem a). (0.5)

3. a) State the assignment problem as an ILP problem. (0.5)

b) Prove König's theorem: Suppose that the matrix $\mathbf{C} = [c_{ij}]$ is the cost matrix for an $n \times n$ assignment problem. Suppose that $\widehat{\mathbf{X}} = [\widehat{x}_{ij}]$ is an optimal solution to this problem. Let $\mathbf{C}'$ be the matrix formed by adding the number α to each entry of one of the rows (or columns) of $\mathbf{C}$. Then $\widehat{\mathbf{X}}$ is an optimal solution of the new assignment problem defined by $\mathbf{C}'$. (0.5)

4. a) What is the maximal flow from node 1 to node 7 in the following network? Give the actual flow on each of the edges as well as the value of the flow. Justify your answer, e.g. by referring to a theorem that you have learnt in the course. (0.7)



b) True or false: For any flow network G and any maximum flow on G, there is always an edge e such that increasing the capacity of e increases the maximal flow of the network. Justify your answer. (0.3)

VAR GOD VÄND! / PLEASE TURN OVER!

5. Solve the following LP problem:

$$\begin{aligned}
&\text{minimize} \quad z = 3x_{11} + 12x_{12} + 8x_{13} + 10x_{21} + 5x_{22} + 6x_{23} + 6x_{31} + 7x_{32} + 10x_{33}\\
&\text{subject to} \quad \begin{cases}
x_{11} + x_{12} + x_{13} = 90,\\
x_{21} + x_{22} + x_{23} = 30,\\
x_{31} + x_{32} + x_{33} = 100,\\
x_{11} + x_{21} + x_{31} \le 70,\\
x_{12} + x_{22} + x_{32} \le 110,\\
x_{13} + x_{23} + x_{33} \le 80,\\
x_{ij} \ge 0.
\end{cases}
\end{aligned}$$

6. You need to find the safest possible route to send a message from station A to station F. You can use the intermediate stations B to E. The probability to lose the information while sending a message between different stations directly is collected in the table below. It is assumed that these events (namely, that the information is lost along the respective direct links) are mutually independent.

From \ To	A	B	C	D	E	F
A	0	0.05	0.15	0.30	0.35	0.45
B	0.06	0	0.10	0.18	0.29	0.38
C	0.16	0.08	0	0.11	0.19	0.28
D	0.26	0.18	0.08	0	0.09	0.20
E	0.36	0.25	0.20	0.07	0	0.02
F	0.45	0.38	0.30	0.21	0.09	0

What is the safest route, and what is the probability that information gets lost along that route? The problem can be solved with a slight modification of one of the algorithms that we learnt in the course. Which algorithm is this, and what type of problem does it (usually) solve?

LYCKA TILL! / GOOD LUCK!