



LUND
UNIVERSITY

Written Examination
Linear Algebra
Saturday, 21 November 2015
Duration: 08:00–13:00

Centre for Mathematical Sciences
Mathematics, Faculty of Science

In order to sit for the examination you must be enrolled in the course. No aids are allowed. Use the papers provided by the department and write on one side only of each page. Fill in the cover completely and write your initials on each paper you hand in. Give concise and short arguments and draw figures when applicable.

1. Find the vector $\mathbf{x} = (x_1, x_2, x_3)$ that minimises the distance between $A\mathbf{x}$ and \mathbf{y} where

$$A = \begin{bmatrix} 0 & 2 & 2 \\ 2 & 3 & -1 \\ 1 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix} \quad \text{and} \quad \mathbf{y} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}.$$

2. A linear transformation $F : \mathbf{R}^3 \rightarrow \mathbf{R}^3$ has the following properties:

$$\begin{aligned} F(1, 1, 3) &= (1, 2, 3), \\ F(0, 2, 1) &= (1, -1, 2), \\ F(0, 0, 1) &= (5, 5, 4). \end{aligned}$$

Find the matrix of F with respect to the standard basis for \mathbf{R}^3 .

3. Find an orthonormal basis for \mathbf{R}^3 for which one of the basis vectors is parallel to the line $(x, y, z) = t(1, 2, 3)$.
4. The matrix of a linear transformation with respect to an orthonormal basis is

$$\frac{1}{11} \begin{bmatrix} 7 & -6 & 6 \\ -6 & 2 & 9 \\ 6 & 9 & 2 \end{bmatrix}.$$

Show that the linear transformation is an orthogonal reflection and find its subspace of reflection.

5. Find the minimum value of $x_1^2 + x_2^2 + x_3^2$ subject to the constraint

$$q(x_1, x_2, x_3) = 7x_1^2 + 3x_2^2 + 7x_3^2 + 2x_1x_2 + 4x_2x_3 = 1.$$

6. Let $\mathbf{u}_1, \dots, \mathbf{u}_k$ be k vectors of equal length in \mathbf{R}^n . Show that the set

$$U = \{\mathbf{u} \in \mathbf{R}^n; \|\mathbf{u} - \mathbf{u}_i\| = \|\mathbf{u} - \mathbf{u}_j\|, 1 \leq i < j \leq k\}$$

is a linear subspace of \mathbf{R}^n .