



LUND
UNIVERSITY

Written Examination
Linear Algebra
Thursday, 29 October 2015
Duration: 08:00–13:00

Centre for Mathematical Sciences
Mathematics, Faculty of Science

In order to sit for the examination you must be enrolled in the course. No aids are allowed. Use the papers provided by the department and write on one side only of each page. Fill in the cover completely and write your initials on each paper you hand in. Give concise and short arguments and draw figures when applicable.

1. Find the parabola $y = ax^2 + bx + c$ that is the best least squares fit to the following points:

$$\begin{array}{cccccc} x & -2 & -1 & 0 & 1 & 2 \\ y & 4 & 2 & 1 & 3 & 5 \end{array}$$

2. A linear transformation $F : \mathbf{R}^3 \rightarrow \mathbf{R}^3$ has the following properties:

$$\begin{aligned} F(1, -2, 3) &= (1, 2, 3), \\ F(1, 2, 3) &= (9, 6, 11), \\ F(0, 0, 1) &= (1, 1, 1). \end{aligned}$$

Find the matrix of F with respect to the standard basis for \mathbf{R}^3 .

3. Find an orthonormal basis for the plane $x + y + 3z = 0$. Extend this basis to an orthonormal basis for \mathbf{R}^3 .
4. Find the maximum and minimum values of the quadratic form

$$q(x_1, x_2, x_3) = x_1^2 + 3x_2^2 + x_3^2 + 2x_1x_2 - 2x_1x_3 - 2x_2x_3$$

subject to the constraint $x_1^2 + x_2^2 + x_3^2 = 1$. Also state the points where they occur.

5. The matrix of a linear transformation with respect to an orthonormal positively oriented basis is

$$\frac{1}{3} \begin{bmatrix} 2 & -1 & 2 \\ 2 & 2 & -1 \\ -1 & 2 & 2 \end{bmatrix}.$$

Show that the linear transformation is a rotation and find the axis and angle of rotation.

6. Let A be an $n \times n$ matrix, \mathbf{x} a vector in \mathbf{R}^n and k a natural number. Show that the vectors $\mathbf{x}, A\mathbf{x}, A^2\mathbf{x}, \dots, A^k\mathbf{x}$ are linearly independent if $A^k\mathbf{x} \neq \mathbf{0}$ and $A^{k+1}\mathbf{x} = \mathbf{0}$.