



LUND
UNIVERSITY

Written Examination
Linear Algebra
Wednesday 8 April 2015
Duration: 8:00–13:00

Centre for Mathematical Sciences
Mathematics, Faculty of Science

In order to sit the examination you must be enrolled in the course. No aids are allowed. Use the paper of the department and write on one page only. Fill in the cover completely and write your initials on every paper you hand in. Give concise and short arguments and draw figures when applicable.

1. State and prove your favorite characterization of the diagonalizability of an $n \times n$ matrix in \mathbb{R}^n .
2. Find bases for the kernel and the range of the linear transformation given by

$$T \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} x_1 - x_2 + x_3 - x_4 \\ x_1 + x_2 + 3x_3 + 3x_4 \\ 2x_1 + x_2 + 5x_3 + 4x_4 \\ 3x_1 + 2x_2 + 8x_3 + 7x_4 \end{pmatrix}$$

3. Find the parabola $y = ax + bx^2$ that is the best least squares fit to the points $(x, y) = (1, 1)$, $(x, y) = (2, 0)$ and $(x, y) = (-1, 1)$.
4. Consider the quadratic form $Q(x, y, z) = x^2 + 2y^2 + 2z^2 - 2xy - 2xz - 2ayz$. For which choices of a is the quadratic form positive definite?
5. Show that if \bar{v} is an eigenvector of a matrix $A \in M_{n \times n}$ and \bar{v} corresponds to a non-zero eigenvalue of A , then $\bar{v} \in \text{Ran}(A)$.
6. Consider the matrix

$$A = \frac{1}{3} \begin{pmatrix} 2 & -1 & a \\ 2 & 2 & b \\ -1 & 2 & c \end{pmatrix}.$$

Find $a, b, c \in \mathbb{R}$ such that A is a rotation. For those values of a, b, c find the axis of rotation and the cosine of the angle of rotation of the matrix A .