LUND UNIVERSITY

Written Examination
Linear Algebra
Saturday 21 March 2015
Duration: 8:00–13:00

Centre for Mathematical Sciences
Mathematics, Faculty of Science

In order to sit the examination you must be enrolled in the course. No aids are allowed. Use the paper of the department and write on one page only. Fill in the cover completely and write your initials on every paper you hand in. Give concise and short arguments and draw figures when applicable.

1. State and prove the dimension theorem.

2. Find bases for the kernel and the range of the linear transformation given by

$$T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} x_1 - x_2 + x_3 - x_4 \\ x_1 + x_2 + 3x_3 + 3x_4 \\ 2x_1 + x_2 + 5x_3 + 4x_4 \\ 3x_1 + 2x_2 + 8x_3 + 7x_4 \end{pmatrix}$$

3. Find the line $y = ax + b$ that is the best least squares fit to the points $(x, y) = (-1, 0)$, $(x, y) = (0, 2)$, $(x, y) = (1, 3)$ and $(x, y) = (2, 6)$.

4. Find the values $k \in \mathbb{R}$ for which the matrix A below is diagonalizable in $\mathbb{R}^3$,

$$A = \begin{pmatrix} k & -2 & 1 \\ 4 & -k & 2 \\ 0 & 0 & 1 \end{pmatrix}.$$

5. Let $\{\bar{v}_1, \bar{v}_2, \ldots, \bar{v}_k\}$ be a collection of linearly independent vectors in $\mathbb{R}^n$ and let $P \in M_{nxn}$ be an invertible matrix. Prove $P\bar{v}_1, P\bar{v}_2, \ldots, P\bar{v}_k$ are linearly independent vectors.

6. Let A be the matrix

$$\begin{pmatrix} -3 & -2 & -6 \\ 6 & -3 & -2 \\ -2 & -6 & 3 \end{pmatrix}.$$

Determine the real number c such that the matrix cA becomes the matrix of a rotation. Find the rotation axis and the cosine of the rotation angle.