



LUNDS
UNIVERSITET

Matematikcentrum
Matematik NF

Fourieranalys
Torsdag den 12 januari 2006
Skrivtid: 08.00–13.00

Svar

1. a)

$$\frac{2}{\pi} \sum_{k=1}^{\infty} \frac{(1 - \cos k) \sin kx}{k}$$

b)

$$\sum_{k=1}^{\infty} \frac{(1 - \cos k)^2}{k^2} = \frac{\pi}{2} \quad \text{och} \quad \sum_{k=1}^{\infty} \frac{(1 - \cos k) \sin k}{k} = \frac{\pi}{4}$$

2.

$$u(x, t) = \sum_{k=0}^{\infty} \frac{8}{\pi(2k+1)^2} \cos((2k+1)x) e^{-(2k+1)^2 t}$$

3. a)

$$\hat{u}(\xi) = \begin{cases} -i\pi e^{-|\xi|}, & \xi > 0, \\ i\pi e^{-|\xi|}, & \xi < 0. \end{cases}$$

b)

$$\int_{-\infty}^{\infty} \frac{x^2}{(1+x^2)^2} dx = \frac{\pi}{2}$$

4.

$$u(x) = \frac{\sin 3x}{x} - \frac{\pi}{1+\pi} \frac{\sin x}{x}$$

5.

$$\sqrt{\frac{\pi}{2}} \frac{2k!}{k!} 2^{-k}$$