



LUND
UNIVERSITY

Written Examination
Fourier Analysis, 7.5 credits
Thursday August 26, 2010
Time: 08.00-13.00

Centre for Mathematical Sciences
Mathematics, Faculty of Science

Use only the distributed paper sheets; write only on one side, and no more than one problem per sheet. Fill in the cover form fully and initialize each sheet. Write legibly. Give clear and brief arguments. Draw a picture if this helps.

1. Let $u(x) = |\sin x|$, $-\pi \leq x \leq \pi$.
 - a) Find the Fourier series of u .
 - b) Find the sum of the series

$$\sum_{k=0}^{\infty} \frac{1}{1 - 4k^2}.$$

2. Find a solution u of the following problem

$$\begin{aligned} \partial_t u(x, t) &= 3\partial_x^2 u(x, t) && \text{when } t > 0 \text{ and } 0 < x < \pi, \\ u(0, t) &= u(\pi, t) = 0 && \text{when } t > 0, \\ u(x, 0) &= \sin^3 x && \text{when } 0 < x < \pi. \end{aligned}$$

3. Find a function u such that

$$u(x) + 5 \int_{-\infty}^{\infty} u(x-y) \frac{\sin y}{y} dy = \frac{\sin 5x}{x}.$$

4. Let $f(x) = \cos(\frac{x}{2})$ for $|x| < \pi$ and $f(x) = 0$ for $|x| \geq \pi$.
 - a) Find the Fourier transform of f .
 - b) Evaluate of the integrals

$$\int_{-\infty}^{\infty} \frac{\cos(\pi\xi)}{1 - 4\xi^2} d\xi \quad \text{and} \quad \int_{-\infty}^{\infty} \frac{\cos^2(\pi\xi)}{(1 - 4\xi^2)^2} d\xi.$$

5. a) Let $f, g, f * g \in L^1(\mathbb{R}) \cap L^2(\mathbb{R})$. Show that

$$\int_{-\infty}^{\infty} f * g(x) dx = \int_{-\infty}^{\infty} f(x) dx \cdot \int_{-\infty}^{\infty} g(x) dx.$$

- b) Suppose that $f \in L^1(\mathbb{R}) \cap L^2(\mathbb{R})$ satisfies $\int_{-\infty}^{\infty} f(x) dx = 1$. For $b > a$, evaluate

$$\int_{-\infty}^{\infty} \int_{x-b}^{x-a} f(y) dy dx.$$