1.
Critical points and their character:
(0,0): saddle,   Q(h,k) = -8hk-2k^2=-2((k+2h)^2-4h^2). Indefinite
(4,0): saddle,   Q(h,k) = 8hk-2k^2 =-2(k-2h)^2+8h^2. Indefinite
(2,-2): local max, Q(h,k) = -4h^2-2k^2. Negative definite.

2.
Solution: pi*(1-exp(-4))/8

3.
a) 0
b) dP/dy-dQ/dx = 0, Sol. e-1/e

4.
g(r) = 1 + c/r with c = 1
Solution f(x,y) = 1 + 1/sqrt(x^2+y^2)

5a. The directional derivate is 16/(9*sqrt(3))
5b. The maximum directional derivative is sqrt(96)/9 which is smaller than 10/9 thus the answer is NO
5c. No limit exists since we can find at least two different values for the limit (it depends on the angle phi).
5d. The function f(x,y) reduces to cos(2*theta) which achieves the maximum of 1 if the angle theta=0 and minimum of -1 if for instance theta=pi/2 for any r (even as r goes to infinity).

6. The radius r = sqrt(2). The c = 2.