

1. This can be solved by reformulating as counting the number of arrangements of \circ s (objects) and $|$ s (dividors).
 - a) We have 12 objects and 2 dividors resulting in $\frac{14!}{12!2!} = 91$ arrangements
 - b) By first giving 4 oranges to Tom we see that there are $\frac{10!}{8!2!} = 45$ such arrangements.
 - c) There are $\frac{10!}{9!1!} = 10$ arrangements where Tom gets exactly three oranges so using **a** we find that there are $91 - 10 = 81$ arrangements where Tom does not get three oranges.
2.
 - a) There are $\frac{9!}{2!2!} = 90720$ such arrangements
 - b) Regarding BOX as a unit we get $\frac{7!}{2!2!} = 1260$ arrangements that are not allowed so the answers is $90720 - 1260 = 89460$ arrangements.
 - c) If we first arrange the consonants, then choose three positions for the vowels and finally choose in which of those positions we should put O we end up with $\frac{6!}{2!} \cdot \binom{7}{3} \cdot 3 = 360 \cdot 35 \cdot 3 = 37800$ possibilities.
3. Solving the characteristic equation $r^2 - r - 2 = 0$ we find the solutions $a_n^{(H)} = A2^n + B(-1)^n$ to the corresponding homogenous equation. There should be a particular solution of the form $a_n^{(P)} = Cn2^n + Dn + E$. Substituting this into the recurrence we find that $a_n^{(P)} = n2^{n-1} - \frac{n}{2} - \frac{3}{4}$ and finally the initial values for for a_n allows us to solve for A and B to get $a_n = a_n^{(H)} + a_n^{(P)} = 2^n + \frac{3}{4}(-1)^n + n2^{n-1} - \frac{n}{2} - \frac{3}{4}$
4. Using the fact that if p and q are primes $x \equiv a \pmod{pq}$ is equivalent to

$$\begin{cases} x \equiv a \pmod{p} \\ x \equiv a \pmod{q} \end{cases}$$

we obtain the following system:

a)

$$\begin{cases} x \equiv 3 \equiv 1 \pmod{2} \\ x \equiv 3 \equiv 0 \pmod{3} \\ x \equiv 5 \equiv 1 \pmod{2} \\ x \equiv 5 \equiv 0 \pmod{5} \end{cases}$$

Applying the method in the proof of the Chinese remainder theorem we find that the solutions are $x = 15 + 30k$ where k is any integer.

b) Using the same method as above we find that the first and third equations have no common solutions so there are no solutions in this case.

5. a) The words have four coefficients in \mathbb{Z}_2 so there are $2^4 = 16$ words.

- b) Turning G into normal form by subtracting the second row from the first we get the control matrix

$$H = \begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

- c) The separation is 4 which can be seen by listing the code words and finding the smallest weight of the code or by noting that any three columns of H are linearly independent but that there are dependencies between four columns.
- d) The words w_1, w_2 and w_3 have the syndromes 0010, 0000 and 0011 respectively. This shows that w_1 is the only code word among them. The unique lowest weight word with the same syndrome as w_1 is $y = 00000010$ so w_1 can be corrected into $w_1 - y = 10100101$. The syndrom of w_3 is not a column of H so it is not the syndrom of any word of weight 1. Of weight 2 there are more than one word with this syndrom, for example 00000011 and 11000000. Therefore w_3 cannot be (uniquely) corrected.
6. a) Yes, commutativity is easily check directly from the multiplication formulas in each case.
- b) One can easily verify that $(1, 1)$ is unity in R_1 and $(0, 1)$ is unity in R_2 .
- c) The ring R_1 has zero divisors: $(0, 1) \odot (1, 0) = (0, 0)$. That R_2 has no zero divisors can be seen verifying that the system

$$\begin{cases} ac + ad + bc = 0 \\ ac + bd = 0 \end{cases}$$

with $a, b, c, d \in \mathbb{Z}_2$ has no solutions except those with either $(a, b) = (0, 0)$ or $(c, d) = (0, 0)$.

- d) R_1 is not a field since fields cannot have zero divisors. R_2 , on the other hand, is a field. It suffices to check that $(1, 0) \otimes (1, 1) = (0, 1)$ to see that any non-zero element has an inverse. (The unity of a ring always has itself as inverse so we do not need to find an inverse for $(0, 1)$.)