LUNDS TEKNISKA HÖGSKOLA
MATEMATIK

TENTAMENSSKRIVNING
DISKRET MATEMATIK
2018–08–25 kl 8–13

No books, notes, computational devices, etc. are allowed. Use clear handwriting and give clear careful motivations. All answers should be simplified, but may contain factorials, binomial coefficients, Stirling numbers and powers in cases where they cannot easily be computed. Fill in the form completely and write your personal identifier on each sheet of paper.

1. Solve the recurrence relation

$$a_{n+2} + 5a_{n+1} + 6a_n = 4(-2)^n, a_0 = 6, a_1 = -8$$

2. Find all integers x satisfying the system of congruencies

$$\begin{cases} x \equiv 2 \pmod 3 \\ x \equiv 1 \pmod 4 \\ x \equiv 3 \pmod 7 \end{cases}$$

3. In how many ways can seven different marbles be distributed into four boxes

a) If all the boxes are distinct?

b) If all boxes are distinct and no box may be left empty?

c) If all boxes are identical and no box may be left empty?

d) If all boxes are identical and at most one box may be left empty?

4. How many non-decreasing sequences of length n in the numbers $1, 2, 3$, where the number 1 occurs an even number of times, are there? (The answer should be an expression in n where the number of terms is fixed, ie does not depend on n.)

5. Consider the linear code C over $\mathbb{Z}_7$ with control matrix

$$H = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{pmatrix}$$

a) What is the dimension of C?

b) What is the separation of C?

c) For each of the words $w_1 = (2\ 0\ 3\ 0\ 4\ 0)$, $w_2 = (1\ 1\ 1\ 0\ 2\ 2)$ and $w_3 = (1\ 1\ 1\ 1\ 1\ 1)$ decide if it is a code word or not. If not decide if it can be corrected and if that is the case compute the corrected word.

6. Let $\mathbb{Z}_{43}^*$ denote the invertible elements of $\mathbb{Z}_{43}$.

a) What are the possible orders of elements in $\mathbb{Z}_{43}^*$?

b) How many elements of each order does $\mathbb{Z}_{43}^*$ contain?