

No books, notes, computational devices, etc. are allowed. Use clear handwriting and give clear careful motivations. All answers should be fully simplified, but may contain factorials. In particular they should not contain binomial coefficients or Stirling numbers. Fill in the form completely and write your personal identifier on each sheet of paper.

1. How many arrangements are there of the letters in COEFFICIENT that do not contain any of the substrings TEN, FIT or CONE?
2. Find all positive integers x satisfying the system of congruencies

$$\begin{cases} 3x \equiv 2 \pmod{7} \\ -x \equiv 10 \pmod{11} \end{cases}$$

3. How many numbers in $[4000, 9999]$ have sum of digits equal to ten?
4. You have an unlimited supply of red $1 \times 1 \times 1$ cm and blue $1 \times 1 \times 2$ cm wooden blocks. Let a_n be the number of colour patterns towers with 1×1 cm base and height n cm can have. Find a recurrence relation for a_n and solve it. (The answer should be an expression for a_n in the variable n .)
5. Consider the linear code C over \mathbb{Z}_7 with control matrix

$$H = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 0 & 1 & 4 & 2 & 2 & 4 & 1 \end{pmatrix}$$

- a) How many words does C contain?
 - b) For each of the words $w_1 = (1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1)$, $w_2 = (1 \ 3 \ 1 \ 1 \ 1 \ 1 \ 1)$, $w_3 = (0 \ -3 \ 1 \ 1 \ 0 \ 1 \ 0)$ and $w_4 = (1 \ -3 \ 1 \ 2 \ 0 \ 1 \ 0)$ decide if it is a code word or not. If not decide if it has only one error and in those cases find the corrected word.
 - c) What is the separation of C ?
6. a) How many irreducible polynomials of degree two are there in $\mathbb{Z}_5[x]$?
b) Is the polynomial $x^4 + 1$ irreducible over $\mathbb{Z}_5[x]$?