LUNDS TEKNISKA HÖGSKOLA
MATEMATIK

TENTAMENSSKRIVNING
DISKRET MATEMATIK
FMAA25/MATB13
2016–08–24 kl 8–13

No books, notes, computational devices, etc. are allowed. Use only paper supplied by the department. Use clear handwriting and give clear careful motivations. All answers should be simplified, but they may contain binomial coefficients, Stirling numbers or factorials. Fill in the form completely and write your personal identifier on each sheet of paper.

1. In how many ways can 12 (identical) oranges be distributed between Tom, John and Jenny

- **a)** without any restrictions?
- **b)** if Tom should have at least four oranges?
- **c)** if Tom should not have three oranges, but could have fewer or more than three?

2. In how many ways can the letters of LETTERBOX be arranged

- **a)** without any restrictions?
- **b)** if the arrangements must not contain the subword BOX?
- **c)** if the arrangements may not have two vowels next to each other?

3. Solve the recurrence relation $a_{n+2} - a_{n+1} - 2a_n = 3 \cdot 2^n + n + 1$ with initial conditions $a_0 = a_1 = 1$.

4. Find all integers x satisfying the system of congruencies

$$\begin{cases} x \equiv a_1 \mod 6 \\ x \equiv a_2 \mod 10 \end{cases}$$

- **a)** in the case $a_1 = 3$ and $a_2 = 5$
- **b)** in the case $a_1 = 2$ and $a_2 = 5$

5. Look at the linear code C over $\mathbb{Z}_2$ with generating matrix

$$G = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$$

- **a)** How many words does C contain?
- **b)** Find a control matrix for C.
- **c)** What is the separation of C?
- **d)** For each of the words $w_1 = (10100111)$, $w_2 = (00111100)$ and $w_3 = (00111111)$ decide if it is a code word or not. If not decide if it is correctable and if it is find the corrected word.

Please turn →

6. On the set M of pairs of elements in $\mathbb{Z}_2$ we introduce the following binary operations: $(a, b) \oplus (c, d) = (a + c, b + d)$, $(a, b) \odot (c, d) = (a \cdot c, b \cdot d)$ and $(a, b) \otimes (c, d) = (a \cdot c + a \cdot d + b \cdot c, a \cdot c + b \cdot d)$. (Here $+$ and $\cdot$ denotes addition and multiplication in $\mathbb{Z}_2$.) We can now define two rings $R_1 = (M, \oplus, \odot)$ and $R_2 = (M, \oplus, \otimes)$. (You may assume here without proving it that the ring axioms are satisfied.) For each of the rings find out the following:

a) Is it commutative?

b) Does it have unity? If so determine the unity.

c) Does it have zero divisors?

d) Is it a field?

Good Luck!