LUND UNIVERSITY

Centre for Mathematical Sciences
Mathematics, Faculty of Science

Written Examination
Algebraic Structures
Saturday April 22, 2017
Duration: 08.00–13.00

Solutions may of course be presented in Swedish! The sheets of the department should be used. Write your initials and at most one solution on each sheet handed in. Give careful motivations to your solutions!

1. Let $\mathrm{SL}(2,\mathbb{R})$ be the set of all 2×2 matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

such that $a, b, c, d \in \mathbb{R}$ and $ad - bc = 1$. Prove that $\mathrm{SL}(2,\mathbb{R})$ is a group under matrix multiplication.

2. Find the multiplicative inverse of $x^2 + 1$ in $\mathbb{Q}[x]/(x^4 - 2)$.

3. Let U_{35} denote the group of units in $\mathbb{Z}_{35}$. Is there a divisor d of $|U_{35}|$ such that no element in U_{35} has order d?

4. Let K be a field such that $\mathbb{Q} \subset K \subset \mathbb{R}$ and $[K : \mathbb{Q}] = 2$. Prove that $K = \mathbb{Q}(\sqrt{d})$ for some integer d.

5. Let n be an integer, $n \geq 3$. Prove that every element of A_n is a product of 3-cycles.

6. Let I be the ideal

$$I = \{f(x)(2x-1) + g(x)(x-2) \mid f(x), g(x) \in \mathbb{Z}[x]\}$$

in $\mathbb{Z}[x]$. Show that $I \neq \mathbb{Z}[x]$.