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OPTIMERING
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Answers and brief comments only for re-exams. No complete solutions.

1. a) No, e.g. $f(t) = -t$, $g(x) = x^2$, $f(g(x)) = -x^2$ not convex.

b) No, GS converges always as the uncertainty intervals shrink exponentially.

c) Yes, a non-empty dual set and unbounded primal problem will contradict the weak duality for LP.

d) No, the Modified Newton method uses $H + \epsilon I$ where H is the matrix of second derivatives.

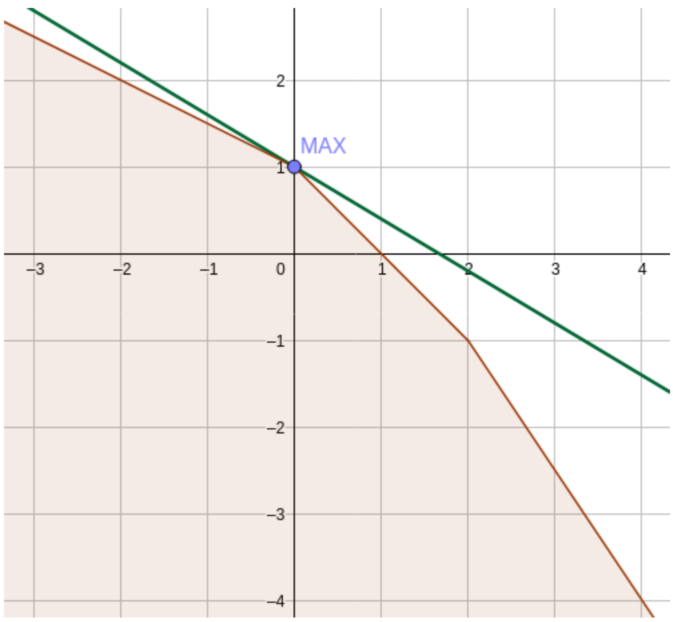
e) Yes, $-\nabla f(x)$ is a descent direction, it does not depend on a line search method.

2. a) Existence: take $(0,0)$. We have $f(0,0) = 1$ and $f(x,y) > 1$ if $x^2+y^2 > 1$. Hence, smaller values than 1 are only in $x^2 + y^2 \le 1$ (compact).
Solve $\nabla f = 0$ to get the minimum at $\pm\frac{1}{2}\big(\sqrt{\ln 2}, \sqrt{\ln 2}\big)$.

b) Newton method converges to the saddle point $(0,0)$ in one step.

3. a) The dual problem is

$$\max 3y_1 + 5y_2 \qquad \text{subject to} \begin{cases} y_1 + 2y_2 & \le 2, \\ y_1 + y_2 & \le 1, \\ 3y_1 + 2y_2 & \le 4. \end{cases}$$



Dual solution: $y = (0,1)$. CSP gives $x_3 = 0$. Hence,

$$\begin{cases} x_1 + x_2 & = 3, \\ 2x_1 + x_2 & = 5 \end{cases} \implies x = (2,1,0).$$

b) Yes, since the linear system

$$\lambda_1(2,1,2) + \lambda_2(1,1,1) + \lambda_3(4,3,2) + \lambda_4(2,1,-1) = (3,2,1), \quad \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 1$$

has the *nonnegative* solution $\lambda = (1/6, 0, 1/2, 1/3)$.

4. a) No, e.g. try the (halv)line $x_2 = x_3 = \ldots = x_n = 0$, $x_1 = t$, $t \geq 0$ and check the second derivative of $\ln(1+t)$.

b) Yes, since $\ln(1 + \|x\|) \leq C$ is equivalent to $\|x\| \leq e^C - 1$ and the function $f(x) = \|x\|$ is convex (easy proof by definition).

c) For $a \notin (0, 2)$ (use Hessian + Sylvester).

5. The set is compact $\Rightarrow$ the minimum exists.
No CQ points. One KKT point: $(1/4, 1/4, -3/4)$, hence, the minimum.

6. a) Formulation: see the book, Corollary 1, p. 265.
The only non-convexity in Problem 5 is the constraint $g_1(x, y, z) = z - x^2 - y^2 \leq 0$. Let's try to make it *inactive* by choosing $u_1 = 0$. This case gives the KKT point from above, hence, it is the global minimum by the sufficient condition (easier as it is no need to check existence, CQ and other KKT cases).

b) See the book, Exercise 6.7, p. 218.

c) See the book, Exercise 3.13, p. 89.