LUNDS TEKNISKA HÖGSKOLA
MATEMATIK

TENTAMENSSKRIVNING
OPTIMERING
2018-08-27 kl 14-19

Students may use the formulae sheet and a pocket calculator. **All solutions must be properly justified**.

1. Decide if the following statements are true/false. Provide a brief explanation or a counter-example. (0.2/item)

a) The composition of two convex functions is convex.

b) The Golden Section line search may diverge if the function is not unimodal.

c) In Linear Programming, if the primal problem is unbounded then the constraints in the dual problem are infeasible (i.e. empty set).

d) The Modified Newton method does not use second derivatives of the function.

e) The Steepest Descent method produces descent search directions for whatever implementation of a line search.

2. Consider the function $f(x, y) = x^2 + y^2 + e^{-4xy}$.

a) Prove that $\min_{(x,y)\in\mathbb{R}^2} f(x, y)$ exists and calculate it. (0.5)

b) Apply the Newton method to f starting from $(x_0, y_0) = (1, 0)$. Verify that the method converges and explain what kind of point you get as an answer (i.e. local/global min/max, saddle point or something else). (0.5)

3. a) Consider the following LP problem

$$\min\,(2x_1 + x_2 + 4x_3) \qquad \text{subject to} \qquad \begin{cases} x_1 + x_2 + 3x_3 &= 3, \\ 2x_1 + x_2 + 2x_3 &= 5, \\ \text{all } x_i \geq 0. \end{cases}$$

State the dual problem, solve the dual problem graphically and use the Complementary Slackness Principle to solve the problem above. (0.6)

b) Does the vector $(3, 2, 1)$ belong to the convex hull of $(2, 1, 2)$, $(1, 1, 1)$, $(4, 3, 2)$ and $(2, 1, -1)$? (0.4)

Please, turn over

4. a) Is the function $\ln(1 + \|x\|)$, $x \in \mathbb{R}^n$, convex? (0.3)

b) Is the set $\{x \in \mathbb{R}^n\colon \ln(1 + \|x\|) \le \text{const}\}$ convex? (0.3)

c) For what $a \in \mathbb{R}$ is the function

$$f(x, y) = \frac{x^a}{y}$$

convex in $\{(x, y)\colon x > 0,\ y > 0\}$? (0.4)

5. Use the KKT necessary condition to solve the optimization problem

$$\min(z - x - y) \quad \text{subject to } z \le x^2 + y^2,\ z \ge 2x^2 + 2y^2 - 1.$$

6. a) State the sufficient KKT condition and explain how it could be used to solve Problem 5 more easily. (0.4)

b) Prove that if g is convex and increasing and h is convex then $g \circ h$ is convex. (0.3)

c) Let H be a symmetric matrix. Prove that any two eigenvectors of H, corresponding to distinct eigenvalues, are H-conjugate. (0.3)

GOOD LUCK!