



LUND
UNIVERSITY

Centre for Mathematical Sciences
Mathematics, Faculty of Science

Written Examination
Flervariabelanalys 1 MATB21
April 8, 2017
Time: 8.00–13.00

Use the papers provided by the department. Write clearly with short and concise motivations. Illustrate with a figure when necessary.

1. Compute the integral

$$\iint_D \frac{x^2 + y^2}{1 + y^2} dx dy,$$

where $D : 0 \leq x \leq y \leq 1$.

2. Find the general (C^2 -smooth) solution $u(x, y)$ to the problem

$$e^{2y} u''_{xx} - u''_{yy} + u'_y = 0.$$

We recommend the change of variables $s = x + e^y$, $t = x - e^y$.

3. Prove that the equation

$$x^y - y^x + 1 = 0$$

defines y as a strictly increasing function of x in some neighbourhood of the point $(1, 2)$.

4. Prove that

$$x^2 + y^2 + \pi \geq 2 + 4 \arctan(xy)$$

for all $(x, y) \in \mathbb{R}^2$.

5. Let $P_0 = (x_0, y_0, z_0)$ be a point on the surface $M : \sqrt{x} + \sqrt{y} + \sqrt{z} = 1$, $x, y, z \geq 0$. Find the equation of the tangent plane to M through P_0 . Find the numbers a, b, c such that $(a, 0, 0)$, $(0, b, 0)$ and $(0, 0, c)$ belongs to the tangent plane. Finally compute the sum

$$a + b + c.$$

Does it depend on the point P_0 ?

6. Compute the integral

$$\iiint_T (x + y + z)(x + y - z)(x - y - z) dx dy dz$$

where T is the domain bounded by the planes $x + y + z = 0$, $x + y - z = 0$, $x - y - z = 0$, and $3x + y - z = 1$.