



LUND  
UNIVERSITY

Written Examination  
Analysis of Several Variables  
Saturday 19 November 2016  
Duration: 8.00–13.00

Centre for Mathematical Sciences  
Mathematics, Faculty of Science

*In order to sit the examination you must be enrolled in the course. No aids except pens, paper and erasers are allowed. Use the paper of the department and write on one page only. Fill in the cover completely and write your initials on every paper you hand in. Give concise and short arguments and draw figures when applicable. The result will be posted at the latest at 12.00 on November 23.*

1. Calculate the integral

$$\iint_T xy \, dx dy$$

where  $T$  is the triangle with corners at  $(0, 0)$ ,  $(1, 2)$  and  $(3, 0)$ .

2. Determine all local extremal points for the function

$$f(x, y) = 3x^2 + 3xy + y^2 + y^3.$$

3. Show that the equation

$$y^3 - 3xy - 8 = 0$$

defines  $y$  as a function of  $x$  in a neighborhood of  $x = 0$  and determine  $y(0)$ ,  $y'(0)$  and  $y''(0)$ .

4. Find the maximum and the minimum of the function

$$f(x, y) = ye^x - xy^2$$

in the set defined by the inequalities

$$0 \leq x \leq 1, \quad 0 \leq y \leq 2$$

5. Calculate the integral

$$\iiint_D \sqrt{x^2 + y^2 + z^2} \, dx dy dz$$

where  $D$  is defined by the inequalities  $x^2 + y^2 + z^2 \leq 1$  and  $z \geq \sqrt{x^2 + y^2}$ .

6. Solve the partial differential equation

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial x \partial y} - 6 \frac{\partial^2 f}{\partial y^2} = 1$$

by using a change of variables

$$u = x + \alpha y, \quad v = x + \beta y,$$

with suitable constants  $\alpha$  and  $\beta$ .