



LUND
UNIVERSITY

Centre for Mathematical Sciences
Mathematics, Faculty of Science

Written Examination
Flervariabelanalys 1 MATB21
April 23, 2016
Time: 8.00–13.00

Use the papers provided by the department. Write clearly with short and concise motivations. Illustrate with a figure when necessary.

1. Find the maximum and the minimum of $f(x, y) = \sqrt{x^2 + y^2} + xy$ over $D : x^2 + y^2 \leq 1$.
2. Prove that the equation

$$\sin xy - \ln(x + y) = 0$$

defines y as a C^2 -smooth function of x in a neighbourhood of the point $(0, 1)$. Also compute the derivative $y'(0)$.

3. Compute the triple integral

$$\iiint_D [y^2 z + x] \, dx dy dz$$

over the domain $D : x^2 - 2x + y^2 + z^2 \leq 0, z \leq 0$.

4. Compute for $x > 1$ the derivative of the function

$$F(x) = \int_{1/x}^1 \frac{\sin(\pi t x / 2)}{t} dt.$$

5. Transform the differential equation

$$x^2 \frac{\partial u}{\partial x} - y^2 \frac{\partial u}{\partial y} = \frac{y^2}{x} + y, \quad x, y > 0$$

by introducing the new variables

$$s = \frac{1}{x} + \frac{1}{y}, \quad t = y.$$

Then solve the differential equation completely.

6. Compute the double integral

$$\iint_D \frac{xy}{x^2 + y^2} \, dx dy$$

where D is the domain in the first quadrant given by the inequalities

$$D : x/\sqrt{3} \leq y \leq x\sqrt{3}, \quad xy \leq 2.$$