LUND UNIVERSITY

Centre for Mathematical Sciences
Mathematics, Faculty of Science

Written Examination
Flervariabelanalys 1 MATB21
March 22, 2016
Time: 8.00–13.00

Use the papers provided by the department. Write clearly with short and concise motivations. Illustrate with a figure when necessary.

1. Determine all local extreme points for the function $f(x, y) = x^3 + 3xy^2 + 2x^2 - y^2$.

2. Prove that the equation

$$x^3 + y^3 + z^3 - xyz = 26$$

defines $z = z(x, y)$ as a C^1-smooth function in a neighbourhood of the point $(1, 1, 3)$. Also show that the function $z(x, y)$ has a critical point at $(x, y) = (1, 1)$.

3. Calculate the triple integral

$$\iiint_D (z^2 + z)\, dxdydz$$

over the domain $D : x^2 + y^2 + z^2 \leq 4,\ z^2 \leq x^2 + y^2$.

4. Transform the differential equation

$$u''_{xx} + u''_{xy} = 0, \qquad x > y > 0$$

by introducing the new variables

$$s = \sqrt{y}, \quad t = \sqrt{x - y}.$$

Then solve the equation completely.

5. Consider the function

$$y(x) = \int_1^{2x} \frac{\ln(xt)}{1 + t^2}\, dt.$$

Compute the derivative $y'(1)$.

6. Compute the arclength of the curve $\gamma : x^{2/3} + y^{2/3} = 1$.