



LUND
UNIVERSITY

Written Examination
Linear Analysis
Saturday April 22, 2017
Duration: 8:00–13:00

Centre for Mathematical Sciences
Mathematics, Faculty of Science

No aids are allowed except the formula sheet provided in the examination hall and pens, pencils and erasers. The result will be posted at the latest on Wednesday, April 26 at 12:00.

1. Which of the following series are convergent?

a) $\sum_{k=1}^{\infty} (-1)^k \cos\left(\frac{1}{k^2}\right)$, b) $\sum_{k=1}^{\infty} \frac{(k!)^3}{(3k)!}$, c) $\sum_{k=1}^{\infty} \frac{(-1)^k}{k + \cos k}$.

2. Find a solution $u(x, t)$ of the following problem:

$$\begin{cases} \partial_t^2 u(x, t) = \partial_x^2 u(x, t), & 0 < x < \pi, \quad t > 0, \\ \partial_x u(0, t) = \partial_x u(\pi, t) = 0, & t > 0, \\ u(x, 0) = \cos 4x \cos 2x, & 0 < x < \pi, \\ \partial_t u(x, 0) = 0, & 0 < x < \pi. \end{cases}$$

3. Let the function u be defined by

$$u(x) = \begin{cases} x, & 0 \leq x < \pi, \\ 0, & -\pi \leq x \leq 0, \end{cases}$$

and $u(x + 2\pi) = u(x)$ for any $x \in \mathbb{R}$.

- a) Find the Fourier coefficients of u .
b) What is the sum of the Fourier series of u for $x = \pi$?
c) Find the sum of the series

$$\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}.$$

4. Find a power series $u(x)$ solving the problem

$$(1 - x^2)u''(x) - 2xu'(x) + 12u(x) = 0, \quad u(0) = 0, \quad u'(0) = 1,$$

and express the answer by means of elementary functions.

5. Let f be the function given by

$$f(x) = \sum_{k=1}^{\infty} \frac{\arctan(kx)}{k^2}$$

for any $x \in \mathbb{R}$.

- a) Show that f is continuous for $x \in \mathbb{R}$.
b) Show that f is differentiable for $x \neq 0$.
c) Show that f is not differentiable for $x = 0$.