



**LUND**  
UNIVERSITY

**Written Examination**  
**Linear Analysis**  
**Thursday March 16, 2017**  
**Duration: 8:00–13:00**

Centre for Mathematical Sciences  
Mathematics, Faculty of Science

**No aids** are allowed except the formula sheet provided in the examination hall and pens, pencils and erasers. The result will be posted at the latest on Wednesday, March 22 at 12:00.

1. Which of the following series are convergent?

a)  $\sum_{k=2}^{\infty} (-1)^k \frac{1}{\ln k}$ .      b)  $\sum_{k=1}^{\infty} \frac{2^k k!}{k^k}$ .      c)  $\sum_{k=1}^{\infty} \frac{(2+3i)^k}{(3+2i)^k}$ .

2. Find a solution  $u(x, t)$  of the following problem:

$$\begin{cases} \partial_t u(x, t) = 3\partial_{xx}^2 u(x, t), & 0 < x < \pi, \quad t > 0, \\ \partial_x u(0, t) = \partial_x u(\pi, t) = 1, & t > 0, \\ u(x, 0) = x + \cos 4x \cos 2x, & 0 < x < \pi. \end{cases}$$

3. Let the function  $u$  be defined by

$$u(x) = e^x \sin x, \quad -\pi \leq x \leq \pi.$$

and  $u(x + 2\pi) = u(x)$  for any  $x \in \mathbb{R}$ .

- a) Find the Fourier series of  $u$ .  
b) What is the sum of the series for  $x = 3\pi/2$ ?  
c) Find the sum of the series

$$\sum_{n=1}^{\infty} \frac{n^2 - 2}{n^4 + 4}.$$

4. Find a power series solution  $u$  of the differential equation

$$u''(x) + xu'(x) + u(x) = 0, \quad u(0) = 1, \quad u'(0) = 0.$$

Determine the radius of convergence and express  $u$  by means of elementary functions.

5. Let  $f$  be the function given by

$$f(x) = \sum_{k=1}^{\infty} \frac{x}{2 + k^3 x}$$

for any  $x \geq 0$ .

- a) Show that  $f$  is continuous for  $x \geq 0$ .  
b) Show that  $f$  is differentiable for  $x > 0$ .  
c) Show that  $f$  does not have a right derivative at  $x = 0$ .