



LUND
UNIVERSITY

Written Examination
Linear Analysis
Saturday May 28 2016
Duration: 8.00–13.00

Centre for Mathematical Sciences
Mathematics, Faculty of Science

In order to sit the examination you must be enrolled in the course. No aids are allowed except the formula sheet provided in the examination hall. Use the paper of the department and write on one page only. Fill in the cover completely and write your initials on every paper you hand in. Give concise and short arguments and draw figures when applicable. The result will be posted at the latest on Thursday, June 2, at 12.00.

1. Which of the following series are convergent?

a) $\sum_{k=1}^{\infty} k \tan\left(\frac{1}{k^3}\right)$, b) $\sum_{k=1}^{\infty} \frac{(k!)^2}{(2k)!}$, c) $\sum_{k=1}^{\infty} \frac{(-1)^k}{\ln k + (-1)^k/k^2}$.

2. Find a solution $u(x, t)$ to the following problem:

$$\begin{cases} \partial_t u(x, t) = 3\partial_{xx}^2 u(x, t), & 0 < x < \pi, \quad t > 0, \\ \partial_x u(0, t) = \partial_x u(\pi, t) = 0, & t > 0, \\ u(x, 0) = (\cos x)^4, & 0 < x < \pi. \end{cases}$$

3. Let the function f be defined by

$$f(x) = \sin(x/2), \quad 0 \leq x \leq 2\pi.$$

and $f(x + 2\pi) = f(x)$ for any $x \in \mathbb{R}$

- a) Find the Fourier series for f .
b) What is the value of the series for $x = -\pi$?
c) Find the value of the series $\sum_{k=1}^{\infty} \frac{1}{k^2 - \frac{1}{4}}$.
d) Find the value of the series $\sum_{k=1}^{\infty} \frac{1}{(k^2 - \frac{1}{4})^2}$.
4. Find a power series solution to the differential equation

$$2xu''(x) + (1 - 2x)u'(x) + u(x) = 0, \quad u(0) = 1,$$

and determine its radius of convergence.

5. Let f be the function given by

$$f(x) = \sum_{k=1}^{\infty} \frac{k^2 x}{1 + k^4 x^2}$$

for any $x \in \mathbb{R}$.

- a) Show that f is continuous for $x > 0$.
b) Determine if f is continuous for $x = 0$.