



LUND
UNIVERSITY

Written Examination
Linear Analysis
Saturday January 23 2016
Duration: 8.00–13.00

Centre for Mathematical Sciences
Mathematics, Faculty of Science

In order to sit the examination you must be enrolled in the course. No aids are allowed except the formula sheet provided in the examination hall. Use the paper of the department and write on one page only. Fill in the cover completely and write your initials on every paper you hand in. Give concise and short arguments and draw figures when applicable. The result will be posted at the latest on Wednesday, January 27 at 12.00.

1. Which of the following series are convergent?

a) $\sum_{k=1}^{\infty} \cos\left(\frac{1}{k}\right)$, b) $\sum_{k=1}^{\infty} \frac{k!}{(2k)!}$, c) $\sum_{k=1}^{\infty} \frac{(-1)^k}{k + \sin k}$.

2. a) Determine the Fourier series of the function

$$f(x) = \cos(x/2), \quad |x| \leq \pi.$$

b) Find the value of $\sum_{k=1}^{\infty} \frac{(-1)^k}{4k^2-1}$.

3. Find a solution $u(x, t)$ to the following problem:

$$\begin{cases} u_t(x, t) = 3u_{xx}(x, t), & 0 < x < \pi, \quad t > 0, \\ u_x(0, t) = u_x(\pi, t) = 0, & t > 0, \\ u(x, 0) = (\sin x)^4, & 0 < x < \pi. \end{cases}$$

4. a) Find a power series solution to the differential equation

$$xu''(x) + (1+x)u'(x) + au(x) = 0, \quad u(0) = 1,$$

when the constant a is equal to 1. Express the answer by means of elementary functions.

- b) Show that the equation has a power series solution for any a and find its radius of convergence.

5. Let

$$f_n(x) = \frac{x}{1 + \frac{x^n}{n}}.$$

- a) Show that the sequence f_n has a pointwise limit,

$$\lim_{n \rightarrow \infty} f_n(x) = f(x),$$

for any real number x and determine $f(x)$.

- b) Does f_n converge to f uniformly on $[0, 1]$?
c) Does f_n converge to f uniformly on \mathbb{R} ?