



LUND  
UNIVERSITY

Written Examination  
Linear Analysis  
Thursday January 7 2016  
Duration: 8.00–13.00

Centre for Mathematical Sciences  
Mathematics, Faculty of Science

**Turn the page for the Swedish translation**

*In order to sit the examination you must be enrolled in the course. No aids are allowed except the formula sheet provided in the examination hall. Use the paper of the department and write on one page only. Fill in the cover completely and write your initials on every paper you hand in. Give concise and short arguments and draw figures when applicable. The result will be posted at the latest on Monday January 11 at 12.00.*

1. Which of the following series are convergent?

a)  $\sum_{k=1}^{\infty} \tan\left(\frac{1}{k}\right),$

b)  $\sum_{k=1}^{\infty} \frac{k^5}{2^k},$

c)  $\sum_{k=1}^{\infty} \frac{(-1)^k}{\ln k + \frac{1}{k}}.$

2. a) The function  $u$  has period  $2\pi$  and

$$u(x) = |x|, \quad |x| \leq \pi.$$

Determine the Fourier series of  $u$ .

- b) Find the value of

$$\sum_{k=0}^{\infty} \frac{1}{(2k+1)^2}.$$

3. Find a solution  $u(x, t)$  to the following problem:

$$\begin{cases} u_t(x, t) = 5u_{xx}(x, t), & 0 < x < \pi, \quad t > 0, \\ u_x(0, t) = u_x(\pi, t) = 0, & t > 0, \\ u(x, 0) = (\sin x)^2 \cos 2x, & 0 < x < \pi. \end{cases}$$

4. Determine a power series  $u(x)$  solving the problem

$$u''(x) + xu'(x) + u(x) = 0, \quad u(0) = 1, \quad u'(0) = 0,$$

and express the answer by means of elementary functions.

5. Set

$$s(x) = \sum_{k=1}^{\infty} \frac{|\sin kx|}{1 + k^2 x^2}.$$

- a) Show that  $s(x)$  is convergent for all real numbers  $x$ .  
b) Show that the function  $s$  is continuous for  $x > 0$  and for  $x < 0$ .  
c) Is  $s$  continuous at 0?