



LUND
UNIVERSITY

Centre for Mathematical Sciences
Mathematics, Faculty of Science

Written Examination
Linear Analysis
Thursday, 20 August 2015
Duration: 8:00–13:00

In order to sit for the examination you must be enrolled in the course. No aids except the formula sheet provided in the examination hall. Use the papers provided by the department and write on one side of each sheet only. Fill in the cover completely and write your initials on each sheet. Write legibly. Give concise and short arguments.

1. Solve the heat conduction problem

$$\begin{aligned}\partial_t u(x, t) &= \partial_x^2 u(x, t), & 0 \leq x \leq \pi, \quad t > 0, \\ u(0, t) &= u(\pi, t) = 0, & t > 0, \\ u(x, 0) &= \sin^3 3x, & 0 \leq x \leq \pi.\end{aligned}$$

2. Which of the following series are convergent?

a) $\sum_{k=1}^{\infty} k \sin\left(\frac{1}{k^3}\right)$, b) $\sum_{k=1}^{\infty} \frac{\cos \pi k}{1 + \sqrt{k}}$, c) $\sum_{k=1}^{\infty} \frac{\sqrt{k}}{1 + k^2} \cos k$.

3. Let u be the 2π -periodic function for which

$$u(x) = \cosh x = \frac{e^x + e^{-x}}{2}$$

when $|x| \leq \pi$.

- a) Find the Fourier series expansion of u .
b) Compute the sum of the series

$$\sum_{k=0}^{\infty} \frac{1}{1 + k^2}.$$

4. Use power series to find a solution of the problem

$$(1 - x^2)y'' - 2xy' + 12y = 0, \quad y(0) = 0, \quad y'(0) = 1.$$

5. Let c be a positive constant. Show that the series

$$\sum_{k=0}^{\infty} (-1)^k x^k (1 - x)^c$$

is uniformly convergent in $[0, 1]$. Hint: The Weierstrass M-test can be applied only when $c > 1$. Marks are given also for correct partial solutions.