



**LUND**  
UNIVERSITY

**Written Examination**  
**Linear Analysis**  
**Friday, 29 May 2015**  
**Duration: 8:00–13:00**

Centre for Mathematical Sciences  
Mathematics, Faculty of Science

*In order to sit for the examination you must be enrolled in the course. No aids except the formula sheet provided in the examination hall. Use the papers provided by the department and write on one side of each sheet only. Fill in the cover completely and write your initials on each sheet. Write legibly. Give concise and short arguments.*

1. Solve the heat conduction problem

$$\begin{aligned}\partial_t u(x, t) &= \partial_x^2 u(x, t), & 0 \leq x \leq \pi, & \quad t > 0, \\ u(0, t) &= u(\pi, t) = 0, & t > 0, \\ u(x, 0) &= x(\pi - x), & 0 \leq x \leq \pi.\end{aligned}$$

2. Let  $u$  be the  $2\pi$ -periodic function for which  $u(x) = x^3 - \pi^2 x$  when  $-\pi \leq x \leq \pi$ .

- a) Find the Fourier series of  $u$ .  
b) Compute the sum of the series

$$\sum_{n=1}^{\infty} \frac{1}{n^6}.$$

3. Compute the sum of the series

$$\sum_{n=1}^{\infty} \frac{n^2 + 2n}{3^n}$$

e.g. by considering a certain power series.

4. Find a power series solution of the problem

$$2xy'' + (1 - 2x)y' + y = 0, \quad y(0) = 1.$$

5. Consider the sequence  $(f_n)_{n=1}^{\infty}$  where

$$f_n(x) = \frac{x^n}{1 + x^{2n}}, \quad x \geq 0.$$

In which of the following intervals is the sequence uniformly convergent?

- a)  $\left[0, \frac{1}{2}\right]$ ,      b)  $\left[\frac{1}{2}, 2\right]$ ,      c)  $[2, \infty)$ .