



LUND
UNIVERSITY

Written Examination
Linear Analysis
Thursday, 8 January 2015
Duration: 8.00–13.00

Centre for Mathematical Sciences
Mathematics, Faculty of Science

In order to sit the examination you must be enrolled in the course. No aids except the formula sheet provided in the examination hall. Use the papers provided by the department and write on one side of each sheet only. Fill in the cover completely and write your initials on each sheet. Write legibly. Give concise and short arguments.

1. Which of the following series are convergent?

a) $\sum_{k=1}^{\infty} e^{-1/k^4}$, b) $\sum_{k=1}^{\infty} (e^{1/k^2} - 1)$, c) $\sum_{k=1}^{\infty} ke^{-k}$.

2. Find a power series solution of the problem

$$2xy''(x) + (1 - 2x)y'(x) - 2y(x) = 0, \quad y(0) = 1.$$

3. Solve the heat conduction problem

$$\begin{aligned} \partial_t u(x, t) &= 4\partial_x^2 u(x, t), & 0 \leq x \leq \pi, \quad t > 0, \\ u(0, t) &= u(\pi, t) = 0, & t > 0, \\ u(x, 0) &= \sin x \cos 3x, & 0 \leq x \leq \pi. \end{aligned}$$

4. Let a be a real number for which $0 < a < 1$ and let u be the function defined in the interval $[0, \pi]$ the graph of which consists of the line segment from the point $(0, 0)$ to the point $(a\pi, 1)$ and the line segment from the point $(a\pi, 1)$ to the point $(\pi, 0)$.

- a) Find the sine series expansion of u .
b) Compute the sum of the series

$$\sum_{n=1}^{\infty} \frac{\sin^2(a\pi n)}{n^4}.$$

5. The functions f_n , $n = 0, 1, 2, \dots$, are defined in the interval $[0, 1]$ and satisfy the recurrence equation

$$f_0(x) = x, \quad f_n(x) = 2f_{n-1}(x) - (f_{n-1}(x))^2, \quad n \geq 1.$$

- a) Show that the sequence $(f_n)_{n=0}^{\infty}$ is pointwise convergent in $[0, 1]$.
b) Is the convergence uniform in the interval $[0, \frac{1}{2}]$?
c) Is the convergence uniform in the interval $[\frac{1}{2}, 1]$?