Written Examination
MATA22 Linear Algebra 1
Thursday 29 October 2015
08:00–13:00

LUND UNIVERSITY

Centre for Mathematical Sciences
Mathematics, Faculty of Science

In order to sit the examination you must be enrolled in the course. No aids allowed. Use the papers provided by the department and write on one side of each sheet only. Fill in the cover completely and write your initials on each sheet. Write legibly. Give concise and short arguments and draw figures when applicable.

1. The vectors $u_1 = (2, 3, 4)$, $u_2 = (1, 2, 3)$ and $u_3 = (3, 2, 2)$ are given with respect to a basis e_1, e_2, e_3 in the 3-dimensional space. Show that u_1, u_2 and u_3 form a basis in the 3-dimensional space and determine the coordinates of e_1, e_2 och e_3 with respect to this new basis.

2. Let ℓ_1 be the intersection line between the planes $x + y - z = 2$ and $x - y + z = -4$. Let M be the plane through the point $(0, 3, -2)$ that is orthogonal to the line ℓ_2 $(x, y, z) = (2t, 1 + 3t, 1 - 3t)$, $t \in \mathbb{R}$. (ON-system assumed.)

a) Show that ℓ_1 is parallel to M and determine the distance between ℓ_1 and M.

b) Do the lines ℓ_1 and ℓ_2 intersect? Motivate your answer!

3. Let S be the sphere of radius 5 centered at the point $(2, 3, -1)$. Let M_1 and M_2 be two parallel planes with normal vector $(1, 2, -2)$, that lie at distance 3 to the centre of the sphere. Let now $\mathcal{C}_1$ and $\mathcal{C}_2$ be the circles of intersection between the sphere and the planes M_1 and M_2 respectively. Determine the equations of the two planes and the midpoints of the circles $\mathcal{C}_1$ and $\mathcal{C}_2$. (ON-system assumed.)

4. Let π be the plane through the point $(2, 1, 1)$ orthogonal to both planes $2x+y+z+1 = 0$ and $x-y+z = 1$. Determine a positively oriented orthonormal basis u_1, u_2, u_3 for the 3-dimensional space such that u_1 and u_2 are parallel to the plane π. Determine the orthogonal projection of the vector $u = (1, 1, 1)$ on u_3. (Positively oriented ON-system assumed.)

5. Let $a \in \mathbb{R}$ and consider the matrix

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & a^3 & a^2 & a \\ 1 & a & 2 & 1 \end{bmatrix}.$$

a) Compute the determinant $D(A)$ and determine those values of a such that A is invertible.

b) Compute the inverse A^{-1} for $a = -1$ and solve the matrix equation $AX + A = E$. (E denotes the unit matrix.)

Please, turn over!

6. Let $P = (3, -1, 2)$, $Q = (-1, 1, 2)$ and $R = (3, 2, -1)$ be the midpoints of the edges AB, BC and AC in a triangle $\triangle ABC$. (Positively oriented ON-system).

a) Show using vector arithmetic that $APQR$ is a parallelogram.

b) Determine a parametric equation of the line ℓ_1 that contains the edge AB.

c) Let N be the midpoint of the segment PQ and consider the line ℓ_2 through N that is orthogonal to the plane of the triangle $\triangle ABC$. Let now S be a point on ℓ_2 that lies at distance 3 to the point N. Compute the volume of the tetrahedron $BPQS$. What is the relationship between the volume of the tetrahedrons $BPQS$ and $BACS$?