Lund University, Numerical Analysis **Spring Term 2, 2019**
Philipp Birken

Finite Volume Methods

Assignment 4

Problem 1
Consider the shallow water equations in the form

$$\mathbf{u}_t + \mathbf{f}(\mathbf{u})_x = \mathbf{0}$$

with $\mathbf{u} = (h, hv)^T$ and

$$\mathbf{f}(\mathbf{u}) = \begin{pmatrix} hv \\ hv^2 + \frac{1}{2}gh^2 \end{pmatrix}.$$

Prove that the matrix

$$\tilde{\mathbf{A}}_{i-1/2} = \begin{pmatrix} 0 & 1 \\ -\tilde{v}^2 + g\bar{h} & 2\tilde{v} \end{pmatrix}$$

is a Roe matrix. Hereby,

$$\tilde{v} = \frac{\sqrt{h_{i-1}}v_{i-1} + \sqrt{h_i}v_i}{\sqrt{h_{i-1}} + \sqrt{h_i}}$$

is the Roe average and $\bar{h} = (h_i + h_{i-1})/2$.

Problem 2
Prove that the flux of the Euler equations as given in class in one dimension is homogenous in the conservative variables $\mathbf{u} = (\rho, \rho v, \rho E)$:

$$\mathbf{f}(\lambda \mathbf{u}) = \lambda \mathbf{f}(\mathbf{u}).$$

Show furthermore that this is not the case in the primitive variables $\mathbf{w} = (\rho, v, p)$.

Problem 3
Consider the shallow water equation as in problem 1. Discretize this using the explicit Euler method with constant mesh width Δt in time and a finite volume method with constant mesh width Δx in space. As a flux function, use Roe's approximate Riemann solver with the entropy fix. Add this to the code you did on the last assignment.

Test the results for

- The Riemann problem with a jump from 2 down to 1 in both components
- The initial data $h_0(x) = 2 + \sin x$, $v_0 = 2$ on $x \in [0, 4]$ and boundary data on the left hand side $h(0,t) = 2 - \sin t$, $v = 2$.

Return: **Tuesday, April 30th, before class**