Lund University, Numerical Analysis **Spring Term 2, 2019**
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Finite Volume Methods

Assignment 3

Problem 1
Consider the following Riemann problems:

$$\mathbf{u}_t + \mathbf{A}\mathbf{u}_x = 0,$$
$$\mathbf{u}(x,0) = \begin{cases} \mathbf{u}_L & x \leq 0, \\ \mathbf{u}_R & x > 0, \end{cases}$$

a)
$$\mathbf{A} = \begin{pmatrix} 0 & 4 \\ 1 & 0 \end{pmatrix}, \quad \mathbf{u}_L = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad \mathbf{u}_R = \begin{pmatrix} 1 \\ 1 \end{pmatrix},$$

b)
$$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 10^{-4} & 2 \end{pmatrix}, \quad \mathbf{u}_L = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad \mathbf{u}_R = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

Compute the solutions, sketch them in the phase plane and sketch $u_1(x,t)$ and $u_2(x,t)$ as functions of x at some fixed time t.

Problem 2
Rewrite the 2D wave equation
$$u_{tt} = c^2(u_{xx} + u_{yy})$$
as a first order system in the variable $q = (v, w, \phi)$ with $v = u_x$, $w = u_y$ and $\phi = u_t$.

Problem 3
Consider the linear system
$$\mathbf{u}_t + \mathbf{A}\mathbf{u}_x = \mathbf{0},$$
Discretize this using the explicit Euler method with constant mesh width Δt in time and a finite volume method with constant mesh width Δx in space. As a flux function, use the Godunov flux.

a) Sketch two different loops to handle the space discretization and compare their differences
b) Program this in a language of your choice and test this on the Riemann problems from problem one on an appropriate domain and visualize the same things as in problem 1.

Return: **Tuesday, April 23rd, in class**