Lund University, Numerical Analysis **Spring Term 2, 2019**
Philipp Birken

Finite Volume Methods

Assignment 1

Problem 1

a) Consider the Navier-Stokes equations in the differential form and assume that $\rho = const$. Simplify the mass and momentum equation under this assumption.

b) Now assume that also the temperature is constant. Do you then still need the energy equation?

Problem 2

Definition 1 *The first order system with* m *equations*

$$\mathbf{u}_t + \mathbf{A}(\mathbf{u})\mathbf{u}_x = \mathbf{0}$$

is called hyperbolic (in the point $(t, \mathbf{x}, \mathbf{u})$ *) if all eigenvalues of* $\mathbf{A}(\mathbf{u})$ *are real and there exist* m *linear independent eigenvectors.*

Prove:

a) The Burgers equation

$$u_t + uu_x = 0,$$

is hyperbolic.

b) The shallow water equation

$$\begin{aligned} v_t + (v^2/2 + \phi)_x &= 0, \\ \phi_t + (v\phi)_x &= 0. \end{aligned}$$

is hyperbolic.

Problem 3

Consider the linear advection equation with discontinous initial data $u_0(x)$. Prove: $u(x,t) = u_0(x - at)$ is a solution of the integral form of the conservation law.

What is the only condition you need to put on $u_0(x)$?

Problem 4
Consider the linear advection equation

$$u_t + u_x = 0, \quad x \in [0,2], t \in [0,\infty],$$

$$u(0,x) = \sin x,$$

$$u(t,0) = \sin t.$$

a) Discretize this using the explicit Euler method with constant time step Δt in time and a finite difference method with constant mesh width Δx in space. Specifically, use the approximation

$$u'(x_i) \approx (u_i - u_{i-1})/\Delta x.$$

   - Program this in a language of your choice and visualize the exact and the numerical solutions for $t = 0$, $t = 1$ and $t = 2$ for $\Delta x = 1/100$ and $\Delta t = 1/200$.
   - Play around with Δx and Δt. What do you observe?

b) Change the space discretization to a central difference

$$u'(x_i) \approx (u_{i+1} - u_{i-1})/(2\Delta x).$$

What do you observe?

Return: **Tuesday, April 2nd, in class**