LUND UNIVERSITY

Centre for Mathematical Sciences
Mathematics, Faculty of Science

Written Examination
Distribution Theory
Monday May 22, 2017
Time: 8.00–13.00

No aids allowed. Use the distributed paper sheets and write only on one side. Fill in the cover sheet completely and initialize every sheet. Write legibly (in Swedish or English). Motivate your conclusions clearly and concisely; draw a picture if that helps to clarify your argument.

Test results: *Posted Tuesday 23 May, before 22.00.*

Oral exams: *On 29–30 May. State your preferences (day and AM/PM) on the cover sheet of your test—at least two options.*

1. Determine the following limits in $\mathcal{D}'(\mathbf{R})$:

a) $\lim_{t\to\infty} t^2 x\cos(tx)$

b) $\lim_{t\to\infty} t^2|x|\cos(tx)$.

2. Determine all $u \in \mathcal{D}'(\mathbf{R})$ that satisfy

$$x^2(u' - 2u) = \delta_0 + \delta_2$$

where δ_a is the Dirac measure at $x = a$

3. For which $f \in \mathcal{S}(\mathbf{R}^n)$ does the convolution equation

$$u - u * f = f * f$$

have a solution $u \in \mathcal{S}(\mathbf{R}^n)$?

4. Compute the Fourier transform of

a) the function $u(x) = \cos(x_1)x_2^3/(1 + x_2^2)$ on $\mathbf{R}^2$

b) the distribution $u \in \mathcal{S}'(\mathbf{R}^2)$ defined by $u(\varphi) = \iint_{|x_1|\le 1} x_2\varphi(x_1 + 1, x_2)\, dx_1 dx_2$.

5. Show that $u_n = \sum_{k=1}^{\infty} k^{-1}\left(\delta_{k/n^2} - \delta_{-k/n^2}\right) \in \mathcal{D}'(\mathbf{R})$ for $n = 1, 2, 3, \ldots$. Show that $\lim_{n\to\infty} u_n$ exists in $\mathcal{D}'(\mathbf{R})$ and determine the limit. (Hint: look at odd and even test functions.)

6. Let $u \in \mathcal{E}'(\mathbf{R})$ and $v = u * u$. Assume that u, v, ∂u, ∂v, $\cdots$, $\partial^m u$, $\partial^m v$ are linearly dependent over $\mathbf{C}$ for some $m \ge 0$. Show that u is a linear combination of δ_0 and its derivatives.