TOPICS IN ANALYSIS

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Course Description

The aim of the course is to broaden as well as deepen the students knowledge and abilities in mathematical analysis. The course will cover topics in analysis which the student has encountered in previous courses and deepen the students knowledge in these areas, as well as cover topics in analysis that (most) students have not previously encountered and thereby broaden the students knowledge.

The contents of the course will include the following:

- **The Euler–Maclaurin summation formula**, which relates sums and integrals. As an application, one can get a very precise form of Stirling’s formula which approximates factorials.
- **Irrational and transcendental numbers**. A number is transcendental if it is not the root of a polynomial equation with integer coefficients. The numbers $\pi$ and $e$ are known to be transcendental, but how does one prove it?
- **The Riemann and Riemann–Stieltjes integrals**. A Riemann integral of a function $f$ on $[0, 1]$ can be seen as the average of $f$ on $[0, 1]$. The Riemann–Stieltjes integral is a generalisation of the Riemann integral, which can be seen as a weighted average of $f$ on $[0, 1]$. Applications are for instance in probability.
- **Fourier series**. The theory of Fourier series encountered in previous courses will be studied in greater depth, including the so-called Dirichlet and Fejér kernels and their application to the study of convergence of Fourier series.
- **Polynomial approximation**. Continuous functions can be approximated by polynomials. This can for instance be done using Bernstein polynomials, which through Bézier curves are important for computer graphics.
- **Important inequalities**. An important part of the analytical tool kit is various inequalities. Several important inequalities such as Jensen’s inequality, Hölder’s inequality, and the arithmetic, geometric and harmonic mean inequality will be studied.

Course Literature: Lecture notes which will be provided.

Examination: Take home exam followed by oral examination.