Two view geometry

- Relative orientation of two cameras
- The epipolar constraints
- The uncalibrated case: The Fundamental Matrix
- The 8-point algorithm
Relative Orientation: Problem Formulation

Given

Two images and corresponding points.

Compute

The structure (3D-points) and the motion (camera matrices).
Mathematical Formulation

Given two sets of corresponding points \( \{x_i\} \) and \( \{\bar{x}_i\} \), compute camera matrices \( P_1 \), \( P_2 \) and 3D-points \( \{X_i\} \) such that

\[
\lambda_i x_i = P_1 X_i
\]

and

\[
\bar{\lambda}_i \bar{x}_i = P_2 X_i.
\]
Ambiguities (uncalibrated case)

Can always apply a projective transformation $H$ to obtain a different solution

$$
\lambda_i x_i = P_1 H H^{-1} x_i = \tilde{P}_1 \tilde{x}_i
$$

and

$$
\bar{\lambda}_i \bar{x}_i = P_2 H H^{-1} x_i = \bar{P}_2 \bar{x}_i.
$$
Simplification

If $P_1 = [A_1 \ t_1]$ and $P_2 = [A_2 \ t_2]$, apply the transformation

$$H = \begin{bmatrix} A_1^{-1} & -A_1^{-1} t_1 \\ 0 & 1 \end{bmatrix}.$$  

Then

$$P_1 H = \begin{bmatrix} A_1 & t_1 \end{bmatrix} \begin{bmatrix} A_1^{-1} & -A_1^{-1} t_1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} I & 0 \end{bmatrix}.$$  

Hence, we may assume that the cameras are

$$P_1 = \begin{bmatrix} I & 0 \end{bmatrix} \text{ and } P_2 = \begin{bmatrix} A & t \end{bmatrix}$$
Consider a single point $x$ in the first image. Any point on the line projects to this point.
Any point on the projection of the 3D line can correspond to $x$. 
Epipolar Geometry
The projected lines should all meet in a point. The so called **epipole** is the projection of the camera center of the other camera.
The epipole $e_1$ is the projection of the $C_2$ in $P_1$. The epipole $e_2$ is the projection of the $C_1$ in $P_2$. $e_1$, $e_2$ usually outside field of view.
See lecture notes.
The Fundamental Matrix

Estimating $F$

If $\mathbf{x}_i$ and $\bar{\mathbf{x}}_i$ corresponding points

$$\bar{\mathbf{x}}_i^T F \mathbf{x}_i = 0.$$ 

If $\mathbf{x}_i = (x_i, y_i, z_i)$ and $\bar{\mathbf{x}}_i = (\bar{x}_i, \bar{y}_i, \bar{z}_i)$ then

$$\bar{\mathbf{x}}_i^T F \mathbf{x}_i = F_{11} \bar{x}_i x_i + F_{12} \bar{x}_i y_i + F_{13} \bar{x}_i z_i + F_{21} \bar{y}_i x_i + F_{22} \bar{y}_i y_i + F_{23} \bar{y}_i z_i + F_{31} \bar{z}_i x_i + F_{32} \bar{z}_i y_i + F_{33} \bar{z}_i z_i$$
JUST DLT IT.
The Fundamental Matrix

Estimating $F$

In matrix form (one row for each correspondence):

\[
\begin{bmatrix}
\bar{x}_1 x_1 & \bar{x}_1 y_1 & \bar{x}_1 z_1 & \ldots & \bar{z}_1 z_1 \\
\bar{x}_2 x_2 & \bar{x}_2 y_2 & \bar{x}_2 z_2 & \ldots & \bar{z}_2 z_2 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\bar{x}_n x_n & \bar{x}_n y_n & \bar{x}_n z_n & \ldots & \bar{z}_n z_n \\
\end{bmatrix}
\begin{bmatrix}
F_{11} \\
F_{12} \\
F_{13} \\
\vdots \\
F_{33}
\end{bmatrix}
= \begin{bmatrix}
0 \\
0 \\
0 \\
\vdots \\
0
\end{bmatrix}
\]

Solve using homogeneous least squares (svd).
$F$ has 9 entries (but the scale is arbitrary). Need at least 8 equations (point correspondences).
The Fundamental Matrix

Issues

Resulting $F$ may not have $\text{det}(F) = 0$.
Pick the closest matrix $A$ with $\text{det}(A) = 0$.

Can be solved using svd, in matlab:

$$[U, S, V] = \text{svd}(F);$$
$$S(3, 3) = 0;$$
$$A = U \ast S \ast V';$$
The Fundamental Matrix

Issues

Normalization needed (see DLT).
If \( x_1 \) and \( \bar{x}_1 \approx 1000 \) pixels, the coefficients \( z_1 \bar{z}_1 = 1 \), \( x_1 \bar{z}_1 = 1000 \) and \( x_1 \bar{x}_1 = 1000000 \). May give poor numerics.

Not normalized:  

Normalized:
The Fundamental Matrix

The 8-point algorithm

- Extract at least 8 point correspondences.
- Normalize the coordinates (see DLT).
- Form $M$ and solve
  \[
  \min_{||v||^2=1} ||Mv||^2,
  \]
  using svd.
- Form the matrix $F$ (ensure that $det(F) = 0$).
- Transform back to the original coordinates.
- Compute a pair of cameras from $F$ (next lecture).
- Compute the scene points (next lecture).
The Fundamental Matrix

Demo.