Hand-in exercise 1 in
Linear and Combinatorial Optimization, 2019.

Complete exercises 1 to 7 below. Work in pairs (two students together) and hand in a solution by sending

(i) the matlab file for the function checkbasic1.m,

(ii) the simplex tableaux for exercises 2 and 3, and

(iii) answers to exercise 4 to 7

to me (sara@maths.lth.se). I expect *one solution per pair of students* with the names of both students clearly written on everything that you hand in.
Due date: 5 February 2019.

1. Construct a MATLAB-script checkbasic1.m with the following form:

```
function [tableau,x,basic,feasible,optimal]=checkbasic1(A,b,c,basicvars);
% [x,basic,optimal,feasible]=checkbasic1(A,b,c,basicvars),
% INPUT: A - mxn matrix
%        b - mx1 matrix
%        c - nx1 matrix
%        basicvars - list of m indices between 1 and n.
% OUTPUT
%        tableau  - the (m+1) x (n+1) matrix representing the simplex tableau
%                    (skip the column corresponding to the objective function z)
%        x        - nx1 matrix. The basic solution corresponding
%                     to basic variables basicvars.
%        basic    - 1 if x is a basic solution
%        optimal  - 1 if x is an optimal solution
%        feasible - 1 if x is a feasible solution
% to the LP-problem in canonical form
%  max z = c'*x
%  subject to A*x=b, x>=0
```

In the following problems in this hand in, you will solve LP problems using the script together with manual simplex steps. Do not write a script for automatically choosing incoming and outgoing variables, since this will be the one of the tasks of lab session 1.

2. Test the script on the following data by manually choosing incoming and outgoing variables, and using your script `checkbasic1.m` for the computations. Compare the tableau you obtain with the correct answer, which can be found on the course homepage.

```
% Script for testing the tableau.
A = [1 1 1 0 0;1 0 0 1 0;8 20 0 0 1];
b = [1;3/4;10];
c = [2;1;0;0;0];

basicvars=[3 4 5];
[tableau,x,basic,feasible,optimal]=checkbasic1(A,b,c,basicvars);
disp('Problem 2a');
```

```
tableau

basicvars=[3 1 5];
[tableau,x,basic,feasible,optimal]=checkbasic1(A,b,c,basicvars);
disp('Problem 2b');
tableau

basicvars=[3 1 2];
[tableau,x,basic,feasible,optimal]=checkbasic1(A,b,c,basicvars);
disp('Problem 2c');
tableau
```

3. Test the script on the following data. Compare the tableau you obtain with the correct answer, which can be found on the course homepage.

```
% Script for testing the tableau.
A = [3 2 1 0 0;5 1 1 1 0;2 5 1 0 1];
b = [1;3;4];
c = [-1;-1;-1;-1;-1]; (converting min-problem to max-problem)

basicvars=[3 4 5];
[tableau,x,basic,feasible,optimal]=checkbasic1(A,b,c,basicvars);
disp('Problem 3a');
tableau

basicvars=[2 4 5];
[tableau,x,basic,feasible,optimal]=checkbasic1(A,b,c,basicvars);
disp('Problem 3b');
tableau
```

4. Solve the problem

$$\max z = x_1 - 2x_2 - 3x_3 - x_4 - x_5 + 2x_6$$

$$\text{subject to} \begin{cases} x_1 + 2x_2 + 2x_3 + x_4 + x_5 = 12 \\ x_1 + 2x_2 + x_3 + x_4 + 2x_5 + x_6 = 18 \\ 3x_1 + 6x_2 + 2x_3 + x_4 + 3x_5 = 24 \\ x_1, x_2, x_3, x_4, x_5, x_6 \geq 0 \end{cases}$$

using the two-phase method.

In phase one, introduce artificial variables and write down matrices `A` and column vectors `b` and `c` for the auxilliary problem. Then solve it using your script `checkbasic1.m`.

In phase two, change the matrices and row vectors by removing the part corresponding to the artificial variables. Start with the basic feasible solution that you found in phase 1, and iterate using your script `checkbasic1.m` until you find an optimal solution.

5. Solve the following linear programming problem using manual simplex steps based on your checkbasic1.m.

$$\max z = 3x_1 + 2x_2 + x_3$$

$$\text{subject to} \begin{cases} 2x_1 - 3x_2 + 2x_3 + x_4 = 3 \\ -x_1 + x_2 + x_3 + x_5 = 5 \\ x_1, x_2, x_3, x_4, x_5 \geq 0 \end{cases}$$

6. Determine the number of basic solutions and basic feasible solutions for the following LP-problem.

$$\max z = x_1 - x_2$$

$$\text{subject to} \begin{cases} x_1 + 2x_2 + x_3 = 4 \\ x_1 + 2x_2 + x_4 = 7 \\ x_1, x_2, x_3, x_4 \geq 0 \end{cases}$$

7. Solve the following problem both by hand and by using the two phase method and your script, and describe what is happening.

$$\max z = \mathbf{c}\mathbf{x},$$

$$\text{subject to} \begin{cases} \mathbf{A}\mathbf{x} = \mathbf{b}, \\ \mathbf{x} \geq \mathbf{0}. \end{cases}$$

where

$$\mathbf{A} = \begin{pmatrix} -1 & 1 \\ -1 & 0 \\ 8 & -20 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 3/4 \\ 10 \end{pmatrix} \quad \text{and } \mathbf{c} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$