⑥ forts.

För att använda skivformeln måste vi uttrycka x som funktion av y (d.v.s. invertera funktionen.) ⑤

$$y = \frac{3x}{\sqrt{1+x^2}} \rightarrow y^2 = \frac{9x^2}{1+x^2} \Leftrightarrow$$

$$(1+x^2)y^2 = 9x^2 \Leftrightarrow y^2 + x^2y^2 = 9x^2 \Leftrightarrow y^2 = (9-y^2)x^2$$

$$\Leftrightarrow x^2 = \frac{y^2}{9-y^2} \Leftrightarrow \underline{\underline{x = \underset{(-)}{+}\frac{y}{\sqrt{9-y^2}}}}$$

(men $x>0 \Leftrightarrow y>0$ och alltså + tecken)

$$x = 1 \Rightarrow y = \frac{3}{\sqrt{2}}$$

Skivformeln $V = \pi \int_0^{3/\sqrt{2}} \frac{y^2}{9-y^2}\, dy =$

$$= \pi \int_0^{3/\sqrt{2}} \left(-1 + \frac{9}{9-y^2}\right) dy = \pi \cdot \left(-\frac{3}{\sqrt{2}} + \frac{3}{2}\int_0^{3/\sqrt{2}} \left(\frac{1}{3-y} + \frac{1}{3+y}\right) dy\right)$$

$$= \pi \cdot \left(-\frac{3}{\sqrt{2}} + \left[\frac{3}{2} \ln\left|\frac{3+y}{3-y}\right|\right]_0^{3/\sqrt{2}}\right) =$$

$$= \pi \left(-\frac{3}{\sqrt{2}} + \frac{3}{2}\left(\ln\left(\frac{3+3\sqrt{2}}{3\sqrt{2}-3}\right) - \underbrace{\ln 1}_{=0}\right)\right) =$$

$$= \pi \left(-\frac{3}{\sqrt{2}} + \frac{3}{2}\ln\underbrace{\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)}_{=\frac{(\sqrt{2}+1)^2}{1}}\right) = 3\pi\left(\ln(1+\sqrt{2}) - \frac{1}{\sqrt{2}}\right).$$

(förläng med konjugatet)

7. Beräkna bågländen av den logaritmiska spiralen som parametriseras som

$$\begin{cases} x(\theta) = e^{-\theta/5}\cos\theta \\ y(\theta) = e^{-\theta/5}\sin\theta . \end{cases}$$

Två alternativa sätt: ① Använd polära koordinater $r = r(\theta)$
② Arbeta med x & y direkt.

Alt 1. $r^2 = x^2 + y^2 = e^{-2\theta/5}(\cos^2\theta + \sin^2\theta) = e^{-2\theta/5}$

$r(\theta) = e^{-\theta/5}$. $\quad r'(\theta) = -\frac{1}{5}e^{-\theta/5}$

$$\text{Kurvlängden} = \int_0^\infty \sqrt{r(\theta)^2 + r'(\theta)^2}\,d\theta =$$

$$= \int_0^\infty \sqrt{e^{-2\theta/5} + \frac{1}{25}e^{-2\theta/5}}\,d\theta = \int_0^\infty e^{-\theta/5}\frac{\sqrt{26}}{5}\,d\theta$$

$$= \frac{\sqrt{26}}{5}\lim_{X\to\infty}\left[-5e^{-\theta/5}\right]_0^X = \frac{\sqrt{26}}{5}\cdot 5 = \sqrt{26}$$

Alt. 2

$$\text{Kurvlängden} = \int_0^\infty \sqrt{x'(\theta)^2 + y'(\theta)^2}\,d\theta =$$

$$= \int_0^\infty e^{-\theta/5}\sqrt{\cos^2\theta + \sin^2\theta}\,d\theta = \int_0^\infty e^{-\theta/5}\,d\theta = \ldots$$

fortsätt som ovan!