Image Analysis - Lecture 4

Energy and Graph based Segmentation

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November 14, 2013

Lecture 4

Contents

- Energy based segmentation
- The Mumford-Shah functional
- Graph cuts
- Mathematical morphology

For first part, see complementary slides.

Mathematical Morphology

Operations on binary images. Can be regarded as non-linear filtering.
$A = \{ (x, y) \in \mathbb{Z}^2 \,|\, f(x, y) = 1 \}$ is considered as a subset of the image.

Definition

Let A and $B \subset \mathbb{Z}^2$.
The **translation** of A with $x = (x_1, x_2) \in \mathbb{Z}^2$ is defined as

$$(A)_x = \{ c \,|\, c = a + x,\ a \in A \} \ .$$

The **reflection** of A is defined as

$$\hat{A} = \{ c \,|\, c = -a,\ a \in A \} \ .$$

Definitions

Definition

The **complement** of A is defined as

$$A^c = \{ c \,|\, c \notin A \} \ .$$

The **difference** of A and B is defined as

$$A - B = \{ c \,|\, c \in A,\ c \notin B \} = A \cap B^c \ .$$

Dilation

Let $B \subset \mathbb{Z}^2$ denote a **structure element**. (Usually B="a circle" with centre at the origin is chosen.)

Definition

The **dilatation** of A with B is defined by

$$A \oplus B = \{ x \,|\, (\hat{B})_x \cap A \neq \emptyset \} \ .$$

■

This can also be written

$$A \oplus B = \{ x \,|\, ((\hat{B})_x \cap A) \subseteq A \} \ .$$

The dilation of A with B can be seen as extending A with B.

Erosion

Definition

The **erosion** of A with B is defined by

$$A \ominus B = \{ x \,|\, (\hat{B})_x \subseteq A \} \quad .$$

■

The erosion of A with B can be seen as diminishing (eroding) A with B.

Opening

Definition

The **opening** of A with B is defined by

$$A \circ B = (A \ominus B) \oplus B \ .$$

Opening = first erosion, then dilation.

- Gives smoother contours.
- Removes narrow passages.
- Eliminates thin parts.

Closing

Definition

The **Closing** of A with B is defined by

$$A \cdot B = (A \oplus B) \ominus B \ .$$

Closing = first dilation, then erosion.

- Gives smoother contours.
- Fills up small parts.
- Fills up holes.

Review - Lecture 4

- Energy based segmentation
  - The Mumford-Shah functional
  - Two-phase Mumford-Shah
  - Statistical interpretation
- Graph cuts
  - Optimization tool with many applications
- Mathematical morphology