APPENDIX B

Standardized wave spectra

Knowledge of which kind of spectral density is suitable to describe sea state data are well established from experimental studies. Qualitative considerations of wave measurements indicate that the spectra may be divided into 3 parts (See Fig. B.1):

1. Sea states dominated by wind sea but significantly influenced by some swell components.

2. More or less pure wind seas or, possibly, swell component located well inside the wind frequency band.

3. Sea states more or less dominated by swell but significantly influenced by wind sea.

One often uses some parametric form of spectral density. Three parametric spectral densities are used in this thesis and will be described in the following sections.

B.1 JONSWAP spectrum

The JONSWAP (JOint North Sea WAve Project) spectrum of Hasselmann et al. (1973) is a result of a multinational project to characterize standardized wave spectra for the Southeast part of the North Sea. The spectrum is valid for not fully developed sea states. However, it is also used to represent fully developed sea states. It is particularly well suited to characterize wind sea when $3.6 \sqrt{H_{m0}} < T_p < 5 \sqrt{H_{m0}}$.

The JONSWAP spectrum is given in the form:

$$S^+(\omega) = \frac{\alpha g^2}{\omega M} \exp \left( - \frac{M}{N} \left( \frac{\omega_p}{\omega} \right)^N \right) \exp \left( \frac{-(\omega/\omega_p-1)^2}{4 \sigma^2} \right)$$

(B.1)
where

\[
\sigma = \begin{cases} 
0.07 & \text{if } \omega < \omega_p \\
0.09 & \text{if } \omega \geq \omega_p
\end{cases} \tag{B.2}
\]

\[M = 5 \quad N = 4 \tag{B.3}\]

\[\alpha \approx 5.061 \frac{H_{m0}^2}{T_p^4} \left(1 - 0.287 \ln(\gamma)\right) \tag{B.4}\]

A standard value for peakedness parameter, \(\gamma\), is 3.3. However, a more correct approach is to relate \(\gamma\) to \(H_{m0}\) and \(T_p\):

\[\gamma = \exp \left(3.484 \left(1 - 0.1975 \left(0.036 - 0.0056 T_p / \sqrt{H_{m0}} \right) T_p^4 / H_{m0}^2 \right)\right) \tag{B.5}\]

Here \(\gamma\) is limited by \(1 \leq \gamma \leq 7\). This parameterization is based on qualitative considerations of deep water wave data from the North Sea, see Torsethaugen et al. (1984); Haver and Nyhus (1986).

The relation between the peak period and mean zero-upcrossing period may be approximated by

\[T_{m02} \approx T_p / \left(1.30301 - 0.01698 \gamma + 0.12102 / \gamma\right) \tag{B.6}\]
The JONSWAP spectrum is identical with the two-parameter Brethschneider, Pierson-Moskowitz, ITTC (International Towing Tank Conference) or ISSC (International Ship and Offshore Structures Congress) wave spectrum, given $H_{m0}$ and $T_p$, when $\gamma = 1$. (See the matlab function jonswap.m in WAFO toolbox) (Brodtkorb et al., 2000; WAFO-group, 2000).

### B.2 Torsethaugen spectrum

Torsethaugen (1993, 1994, 1996) proposed to describe bimodal spectra by

$$
S^+(\omega) = \sum_{i=1}^{2} S_j^+(\omega; H_{m0,i}, \omega_{p,i}, \gamma_i, N_i, M_i, \alpha_i) \tag{B.7}
$$

where $S_j^+$ is the JONSWAP spectrum defined by Eq. (B.1) and (B.2). $H_{m0,i}$, $\omega_{p,i}$, $N_i$, $M_i$ and $\alpha_i$ for $i = 1, 2$ are the significant wave height, angular peak frequency, spectral shape and normalization parameters for the primary and secondary peak, respectively.

These parameters are fitted to 20,000 spectra divided into 146 different classes of $H_{m0}$ and $T_p$ obtained at the Statfjord field in the North Sea in the period from 1980 to 1989. The measured $H_{m0}$ and $T_p$ values for the data ranges from 0.5 to 11 meters and from 3.5 to 19 sec, respectively.

Given $H_{m0}$ and $T_p$ these parameters are found by the following steps:

The distinction between wind dominated and swell dominated sea states is defined by the fully developed sea where

$$
T_p = T_f = 6.6 H_{m0}^{1/3} \tag{B.8}
$$

If $T_p \leq T_f$, the local wind sea dominates the spectral peak, otherwise it is dominated by the swell. For each of the types a non-dimensional period scale is introduced by

$$
\epsilon_{lu} = \frac{T_f - T_p}{T_f - T_{lu}} \tag{B.9}
$$

where

$$
T_{lu} = \begin{cases} 
2 \sqrt{H_{m0}} & \text{if } T_p \leq T_f \text{ (Lower limit)} \\
25 & \text{if } T_p > T_f \text{ (Upper limit)}
\end{cases} \tag{B.10}
$$

defines the lower or upper value for $T_p$. The significant wave height for each peak is given as

$$
H_{m0,1} = R_{pp} H_{m0} \quad H_{m0,2} = \sqrt{1 - R_{pp}^2} H_{m0} \tag{B.11}
$$
where
\[ R_{pp} = (1 - A_{10}) \exp \left( -\frac{\epsilon_{iw}}{A_{1}} \right)^2 + A_{10} \]  
(B.12)

\[ A_{1} = \begin{cases} 0.5 & \text{if } T_p \leq T_f \\ 0.3 & \text{if } T_p > T_f \end{cases} \]

\[ A_{10} = \begin{cases} 0.7 & \text{if } T_p \leq T_f \\ 0.6 & \text{if } T_p > T_f \end{cases} \]  
(B.13)

The primary and secondary peak periods are defined as
\[ T_{p,1} = T_p \]  
(B.14)

\[ T_{p,2} = \begin{cases} T_f + 2 & \text{if } T_p \leq T_f \\ \frac{M_2(N_2/M_2)^{(N_2-1)/M_2}/(1-(N_2-1)/M_2)}{1.28(0.4)^{N_2}}(1-\exp \left( -H_{m0,3/3} \right))^{1/(N_2-1)} & \text{if } T_p > T_f \end{cases} \]  
(B.15)

where the spectral shape parameters are given as
\[ N_1 = N_2 = 0.5 \sqrt{H_{m0}} + 3.2 \]  
(B.16)

\[ M_i = \begin{cases} 4 \left( 1 - 0.7 \exp \left( -\frac{H_{m0}}{3} \right) \right) & \text{if } T_p > T_f \text{ and } i = 2 \\ 4 & \text{otherwise} \end{cases} \]  
(B.17)

The peakedness parameters are defined as
\[ \gamma_1 = 35 \left( 1 + 3.5 \exp \left( -H_{m0} \right) \right) \gamma_T \]  
(B.18)

\[ \gamma_2 = 1 \]  
(B.19)

where
\[ \gamma_T = \begin{cases} \left( \frac{2\pi H_{m0,1}}{gT_p^2} \right)^{0.857} & \text{if } T_p \leq T_f \\ \left( 1 + 6 \epsilon_{iw} \right) \left( \frac{2\pi H_{m0}}{gT_f^2} \right)^{0.857} & \text{if } T_p > T_f \end{cases} \]  
(B.20)

Finally the normalization parameters \( \alpha_i \) \( (i = 1, 2) \) are found by numerical integration so that
\[ \int_0^\infty S_j^+(\omega; H_{m0,i}, \omega_{p,i}, \gamma_i, N_i, M_i, \alpha_i) \, d\omega = H_{m0,i}^2/16 \]  
(B.21)

Preliminary comparisons with spectra from other areas indicate that the empirical parameters given in Eq. (B.13) can be dependent on geographical location. This spectrum is implemented as a matlab function `torsethaugen.m` in the WAFO toolbox (Brodtkorb et al., 2000; WAFO-group, 2000).
B.3 Ochi-Hubble spectrum

Ochi and Hubble (1976) suggested to describe bimodal spectra by a superposition of two modified Pierson-Moskowitz spectra:

\[
S^+(\omega) = \frac{1}{4} \sum_{i=1}^{2} \left( \frac{(\lambda_i + 1/4) \omega_{p,i}^2}{\Gamma(\lambda_i)} \right)^{\lambda_i} \frac{H_{m0,i}^2}{\omega^{4\lambda_i} + 1} \exp\left( -\frac{(\lambda_i + 1/4) \omega_{p,i}^2}{\omega^4} \right)
\]  

(B.22)

where \( H_{m0,i} \), \( \omega_{p,i} \) and \( \lambda_i \) for \( i = 1, 2 \) is the significant wave height, angular peak frequency and spectral shape parameter for the low and high frequency components of the seas, respectively.

The values of these parameters are determined from a analysis of data obtained in the North Atlantic. The source of the data is the same as that for the development of the Pierson-Moskowitz spectrum, but analysis is carried out on over 800 spectra including those in partially developed seas and those having a bimodal shape. In contrast to the JONSWAP and Torsethaugen spectra which is parameterized as function of \( H_{m0} \) and \( T_p \), Ochi and Hubble (1976) gave from a statistical analysis of the data, a family of wave spectra consisting of 11 members generated for a desired sea severity (\( H_{m0} \)) with the coefficient of 0.95.

The values of the six parameters as a function of \( H_{m0} \) are given as:

\[
H_{m0,1} = R_{p,1} H_{m0}
\]  

(B.23)

\[
H_{m0,2} = \sqrt{1 - R_{p,1}^2 H_{m0}}
\]  

(B.24)

\[
\omega_{p,i} = a_i \exp\left( -b_i H_{m0}\right)
\]  

(B.25)

\[
\lambda_i = c_i \exp\left( -d_i H_{m0}\right)
\]  

(B.26)

where \( d_1 = 0 \) and the remaining empirical constants \( a_i, b_i \) (\( i = 1, 2 \)) and \( d_2 \) are given in Table B.1. (See also the matlab function ohspec2.m in WAFO toolbox (Brodtkorb el al., 2000; WAFO-group, 2000).)

Member no. 1 given in Table B.1 defines the most probable spectrum, while member no. 2 to 11 defines the 0.95 percent confidence spectra.

A significant advantage of using a family of spectra for design of marine systems is that one of the family members yields the largest response such as motions or wave induced forces for a specified sea severity, while another yields the smallest response with confidence coefficient of 0.95.

Rodriguez and Guedes Soares (2000) used the Ochi-Hubble spectrum with 9 different parameterizations representing 3 types of sea state categories: swell dominated (a), wind sea dominated (b) and mixed wind sea and swell system with comparable energy (c). Each category is represented by 3 different inter-modal distances between
the swell and the wind sea spectral components. These three subgroups are denoted by I, II and III, respectively. The exact values for the six parameters are given in Table B.2. (See the matlab function ohspec3.m in WAFO toolbox (Brodtkorb et al., 2000; WAFO-group, 2000)).

<table>
<thead>
<tr>
<th>Member no.</th>
<th>$R_{p,1}$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$b_1$</th>
<th>$b_2$</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$d_2$</th>
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<tbody>
<tr>
<td>1</td>
<td>0.84</td>
<td>0.70</td>
<td>1.15</td>
<td>0.046</td>
<td>0.039</td>
<td>3.00</td>
<td>1.54</td>
<td>0.062</td>
</tr>
<tr>
<td>2</td>
<td>0.84</td>
<td>0.93</td>
<td>1.50</td>
<td>0.056</td>
<td>0.046</td>
<td>3.00</td>
<td>2.77</td>
<td>0.112</td>
</tr>
<tr>
<td>3</td>
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<td>0.41</td>
<td>0.88</td>
<td>0.016</td>
<td>0.026</td>
<td>2.55</td>
<td>1.82</td>
<td>0.089</td>
</tr>
<tr>
<td>4</td>
<td>0.84</td>
<td>0.74</td>
<td>1.30</td>
<td>0.052</td>
<td>0.039</td>
<td>2.65</td>
<td>3.90</td>
<td>0.085</td>
</tr>
<tr>
<td>5</td>
<td>0.84</td>
<td>0.62</td>
<td>1.03</td>
<td>0.033</td>
<td>0.038</td>
<td>2.60</td>
<td>0.53</td>
<td>0.069</td>
</tr>
<tr>
<td>6</td>
<td>0.95</td>
<td>0.70</td>
<td>1.50</td>
<td>0.046</td>
<td>0.046</td>
<td>1.35</td>
<td>2.48</td>
<td>0.102</td>
</tr>
<tr>
<td>7</td>
<td>0.65</td>
<td>0.61</td>
<td>0.94</td>
<td>0.039</td>
<td>0.036</td>
<td>4.95</td>
<td>2.48</td>
<td>0.102</td>
</tr>
<tr>
<td>8</td>
<td>0.90</td>
<td>0.81</td>
<td>1.60</td>
<td>0.052</td>
<td>0.033</td>
<td>1.80</td>
<td>2.95</td>
<td>0.105</td>
</tr>
<tr>
<td>9</td>
<td>0.77</td>
<td>0.54</td>
<td>0.61</td>
<td>0.039</td>
<td>0.000</td>
<td>4.50</td>
<td>1.95</td>
<td>0.082</td>
</tr>
<tr>
<td>10</td>
<td>0.73</td>
<td>0.70</td>
<td>0.99</td>
<td>0.046</td>
<td>0.039</td>
<td>6.40</td>
<td>1.78</td>
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</tr>
<tr>
<td>11</td>
<td>0.92</td>
<td>0.70</td>
<td>1.37</td>
<td>0.046</td>
<td>0.039</td>
<td>0.70</td>
<td>1.78</td>
<td>0.069</td>
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Table B.1: Empirical parameter values for the Ochi-Hubble spectral model.

<table>
<thead>
<tr>
<th>Sea state type</th>
<th>Sea state group</th>
<th>$H_{m0,1}$</th>
<th>$H_{m0,2}$</th>
<th>$\omega_{p,1}$</th>
<th>$\omega_{p,2}$</th>
<th>$\lambda_1$</th>
<th>$\lambda_2$</th>
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<tr>
<td>a</td>
<td>I</td>
<td>5.5</td>
<td>3.5</td>
<td>0.440</td>
<td>0.691</td>
<td>3.0</td>
<td>6.5</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>6.5</td>
<td>2.0</td>
<td>0.440</td>
<td>0.942</td>
<td>3.5</td>
<td>4.0</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>5.5</td>
<td>3.5</td>
<td>0.283</td>
<td>0.974</td>
<td>3.0</td>
<td>6.0</td>
</tr>
<tr>
<td>b</td>
<td>I</td>
<td>2.0</td>
<td>6.5</td>
<td>0.440</td>
<td>0.691</td>
<td>3.0</td>
<td>6.5</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>2.0</td>
<td>6.5</td>
<td>0.440</td>
<td>0.942</td>
<td>4.0</td>
<td>3.5</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>2.0</td>
<td>6.5</td>
<td>0.283</td>
<td>0.974</td>
<td>2.0</td>
<td>7.0</td>
</tr>
<tr>
<td>c</td>
<td>I</td>
<td>4.1</td>
<td>5.0</td>
<td>0.440</td>
<td>0.691</td>
<td>2.1</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>4.1</td>
<td>5.0</td>
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<td>0.974</td>
<td>2.1</td>
<td>2.5</td>
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Table B.2: Target spectra parameters for mixed sea states.
References


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